Weibul Texture Filter (WTF) for SAR images

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WEIBULL MULTIPLICATIVE MODEL

The Weibull-distributed random variable x with form parameter $\gamma_x>0$ and scale parameter $\beta_x>0$, has a probability density function given by:

$$f_x(x)=\frac{\gamma_x}{\beta_x}\left(\frac{x}{\beta_x}\right)^{\gamma_x-1}\exp\left[-\left(\frac{x}{\beta_x}\right)^{\gamma_x}\right] \tag{1}$$

The m-order moment can be expressed as,

$$E[x^m]=m\,\beta_x^m\,\Gamma(m/\gamma_x)/\gamma_x \tag{2}$$

For $\gamma_x=2$, the Weibull distribution becomes a Rayleigh distribution, for $\gamma_x=1$, it becomes an Exponential distribution. It can be shown that x^a with $a > 0$ is also Weibull-distributed. If,

$$z=x^a \tag{3}$$

follows that, $f_z(z)=\dfrac{f_x(z^{1/a})}{a\,(z^{1/a})^{a-1}}$ or

$$f_z(z)=\frac{\gamma_z}{\beta_z}\left(\frac{z}{\beta_z}\right)^{\gamma_z-1}\exp\left[-\left(\frac{z}{\beta_z}\right)^{\gamma_z}\right] \tag{4}$$

with form and scale parameters given, respectively, by:

$$\gamma_z=\gamma_x/a \tag{5}$$

$$\beta_z=\beta_x^a \tag{6}$$

Consider b, with $a>b>0$ in a such way that:

$$z=x^a=x^b\,x^{a-b}=\frac{x^b}{E[x^b]}E[x^b]x^{a-b}=s\,t \tag{7}$$

where,

$$s=x^b/E[x^b] \tag{8}$$

$$t=x^{a-b}\,E[x^b] \tag{9}$$

In this form, it is possible to express z as a multiplication of s by t, where s is the speckle, with unitary mean and t is the texture of the Weibull-distributed variable z. The texture t has Weibull distribution with form and scale parameter given, respectively, by:

$$\gamma_t=\gamma_x/(a-b) \tag{10}$$

$$\beta_t=E[x^b]\,\beta_x^{a-b} \tag{11}$$

and the speckle s has Weibull distribution with form and scale parameter given, respectively, by:

$$\gamma_s=\gamma_x/b \tag{12}$$

$$\beta_s=\beta_x^b/E[x^b] \tag{13}$$

Using (2) with $m=b$ and the relation between z and x defined by (3), the texture in (9) can be expressed in terms of γ_x and β_x as,

$$t=b\beta_x^b\Gamma(b/\gamma_x)\,z^{1-b/a}/\gamma_x \tag{14}$$

Using (5) and (6), equation (14) can be re-arranged as,

$$t=p\,\beta_z^p\,\Gamma(p/\gamma_z)\,z^{1-p}/\gamma_z \tag{15}$$

where the factor p is given by

$$p=b/a<1 \tag{16}$$

Equation (15) can be used for the estimation of the texture in the image and t can be considered as the filtered image. For each pixel in the Weibull-distributed image, γ_z and β_z in (15) are locally estimated in a window of dimension NxN surrounding the pixel to be filtered.

It is not necessary to impose the condition that the speckle noise is Rayleigh or Exponential-distributed as it is assumed in one-look amplitude and intensity SAR images, respectively, or any other usual distribution for the multi-look images.

THE FILTERING FACTOR *p*

The factor p ($0<p<1$) gives the filtering intensity. If p is close to one, then $a \approx b$ and the texture t is constant (high filtering). If p is close to zero then $a>>b$ and $t \approx z$ (low filtering). Fig. 1 shows the effect of this parameter in a one-look amplitude SAR-580 L-band image of an area in Freiburg/Germany for different values of p. The Weibull distribution parameters γ_z and β_z were estimated in a window of 9x9.

Alternatively the factor p can be calculated adaptively as follow:

- The form parameter γ_s of speckle is estimated in a homogeneous region of the image to be filtered, or it can be set, for instance, as the mean or mode of γ_z in the whole image.
- Using (12) it can be obtained that $\gamma_x=\gamma_s\,b$.
- Through (5) and (16) p can be calculated adaptively as a function of γ_z that is estimated locally:

$$p=\gamma_z/\gamma_s \tag{17}$$

We note that $t=x^{a-b}\,E[x^b]$ is Weibull-distributed only if $a>b$, and therefore $p<1$ and $\gamma_z<\gamma_s$.

With (17) in (15) the texture becomes:

$$t=\beta_z^{\gamma_z/\gamma_s}\,\Gamma(1/\gamma_s)\,z^{1-(\gamma_z/\gamma_s)}/\gamma_s \tag{18}$$

In (18), it is noticed that if $\gamma_z\to\gamma_s$ then $t\to E[z]$ and, therefore, there is a stark filtering in the image. If $\gamma_z<<\gamma_s$, there is a weak filtering and, if $\gamma_z > \gamma_s$ equation (18) holds, but the texture *t* is not Weibull-distributed anymore.

The factor p can also be known a priori:

- For one-look Amplitude image the speckle is Rayleigh distributed and $\gamma_s=2$, therefore (18) becomes:

$$t = \sqrt{\beta_z^{\gamma_z}\pi}\; z^{1-(\gamma_z/2)}/2 \tag{19}$$

- For one-look Intensity image the speckle is Exponential distributed and $\gamma_s=1$, therefore (18) becomes:

$$t = \beta_z^{\gamma_z} z^{1-\gamma_z} \tag{20}$$

Fig. 2 shows the original SAR-580 X-band, amplitude one-look image, of an area of Freiburg-Germany (theoretically $\gamma_s=2$) and the filtered images, according to (18).

In Fig. 2, it can be observed that a small value of γ_s produces a more effective filtering in the image.

In Figs. 1 and 2 the Weibull distribution parameter γ_z and β_z were estimated, for every pixel, using a window of 9x9 surrounding the pixel.

THE FILTER IMPLEMANTION

The adaptive WTF (Weibull texture Filter) implemented in IDL is given by the program:

wei_tex_filter, image_in, image_out, window_size

where:

- image_in is the original image to be filtered (format float or integer)
- image_out is the filtered output image (format float)
- window_size is the filter window size dimension. It must be odd.

The WTF implemented in the **wei_tex_filter.pro** performes an adaptive filtering using equation (18).

The WTF can be also be implemented in a non-adaptive version through equation (15). In this version you choose the filter window size and also the filtering level p. The filtering level p can be choose between 0 and 1. Zero level means a light filtering and One a stark filtering.

The non adaptive WTF (Weibull texture Filter) implemented in IDL is given by the program:

wei_tex_filter_non_adapt, image_in, image_out, window_size, gain

where:

- gain is the filtering level (0< gain < 1).

Both versions can be obtained in IDL by David Fernandes, david@ele.ita.br.

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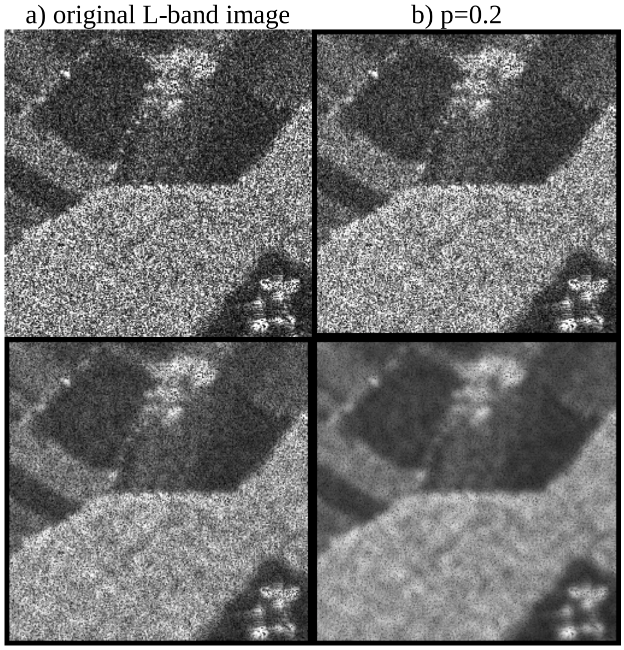
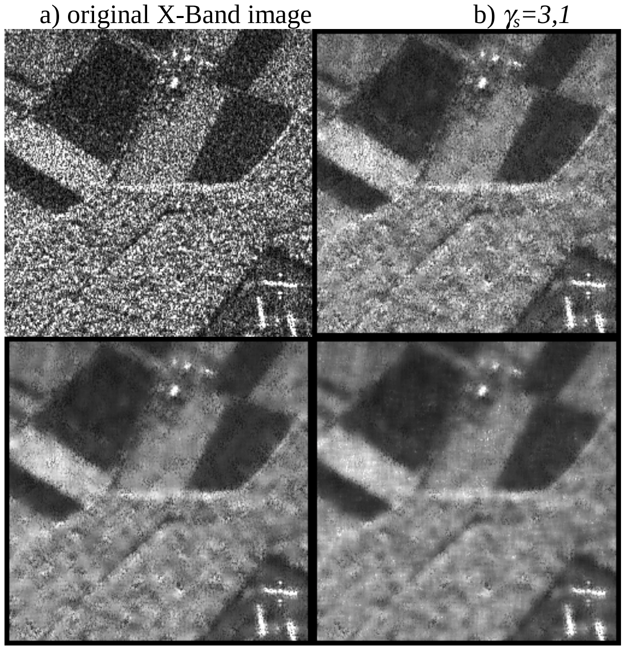
a) original L-band image b) p=0.2



c) p=0.5 d) p=0.8

Figure 1: the effect of *p* in a SAR-580 L-Band image.

a) original X-Band image b) $\gamma_s=3,1$



c) $\gamma_s=2.4$ d) $\gamma_s=2.0$

Figure 2: the effect of γ_s in a SAR-580 X Band image.