EXIT chart optimization of Repeat-Accumulate codes for an N-Frequency T-User Multiple Access Channel with noise

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***Resumo*— Neste artigo mostramos como um sistema baseado em grafos de fatores pode ser projetado para um sistema N-FSK compartilhado por T usuários. O canal possui desvanecimento rápido seletivo em frequência e ruido aditivo Gaussiano branco. O sistema não sabe e não determina quanto de desvanecimento ocorreu num dado instante. O receptor é composto por um detector multiusuário e T decodificadores Repeat-Accumulate (RA), um para cada usuário. Os decodificadores são otimizados conjuntamente e consequentemente possuem os mesmos parâmetros. Uma variação na implementação dos codificadores/decodificadores é suficiente para permitir o acesso múltiplo e codificação de canal, fornecendo taxas próximas da capacidade do canal.**

***Palavras-Chave*— Canais de acesso múltiplo, grafos de fatores, codificação de canal**

***Abstract*— In this paper we show how a graph-based system can be designed for an N-FSK T-user multiple access channel with noise. The channel is a frequency selective fast fading channel with additive white Gaussian Noise. The system does not know or attempt to determine the fading incurred at a given moment. The system is composed of a multiuser detector and T different Repeat-Accumulate (RA) decoders. The decoders are optimized as a set and consequently have the same parameters. A structural variation between the users‘ encoders/decoders is sufficient to allow for multiple access and channel error correction, while providing rates close to channel capacity.**

***Keywords*— Multiple access channels, factor graphs, channel coding.**

I. Introduction

Wideband channels are an option when high data rate and multiple users are desirable. However, for this type of channel, fading is seldom flat. One possible solution is to use a fast frequency hopped code division multiple access (FFH-CDMA) system [1]. At the core of this system is an N-frequency T-user multiple access channel[2], [3]. The sum capacity for this channel, with noise, was presented in [4]. A drawback of traditional FFH-CDMA systems is that its hopping code with length L, used to mitigate the multiuser interference, has spectral efficiency equivalent to a repetition code with rate $1/L$. This results in a sum rate that is considerably lower than the channel's sum capacity.

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In this paper we show that it is possible to transmit information using this channel with rates close to sum capacity without the need of hopping patterns. For this, all users separately encode their messages using a Repeat-Accumulate (RA) encoder which is optimized to match a multiuser detector. All encoders are similar: they have the same parameters, block length and rate. Encoders differ in how the inner node layers for the encoder (and decoder) are connected. The set of parameters that define the encoders (and decoders) can be found using EXIT charts[5]. To do so, an iterative multiuser detector (MUD) was obtained in the form of a factor graph. EXIT curves for the MUD were obtained by simulation. EXIT curves for the decoders were obtained by combining the curves of each of the decoders node layers, resulting in a single curve for the whole decoder. Since all decoders share the same set of parameters, they can be optimized as single entity. Simulations for some proposed systems show that it is possible to achieve very low bit error probability at transmission rates close to the sum capacity, with rates equally distributed among users.

This paper is organized as follows: section II describes the system; section III describes how the MUD factor graph was obtained and how its EXIT charts were generated; section IV shows how the encoders were optimized and constructed; section V presents simulation results and in section VI final remarks close this paper.

Some remarks about mathematical notation: $P()$ is a probability mass function (p.m.f) and $p()$ is a probability density function (p.d.f.). For both cases, function arguments are sufficient to identify them. Bold letters are vectors or matrixes.

II. Multiple Access system description

The model developed here was shown in [4] based on the work from [1] and [3].

There are T users that share one fast fading channel. The channel is divided into $N = 2^K$ subchannels with bandwidth τ^{-1} each, where K is an integer. The fading in each of these subchannels is considered flat and independent from other subchannels. The frequency-time window of one subchannel during a duration of time τ is a chip.

During a given time frame with duration $(B/K)\tau$, each user independently chooses a set of I information bits. These bits are encoded into a block of B transmission bits using a systematic RA encoder. It is assumed that all users have encoders with the same design parameters but with different implementations. After interleaving, the B transmission bits per user are grouped into sets of K bits. The following describes how one set of K bits per user results in the signal received by the MUD.

At a given time instant, K bits $\{b_0^j, b_1^j, ..., b_{K-1}^j\}$ from the j^{th} user are mapped into a message m^j from a set of N possible messages $\{0, 1, ..., N-1\}$. With no loss of generality, the mapping $m^j = \sum_{k=0}^{K-1} b_k 2^K$ can be assumed. The superscript ^{j} identifies the user. Let E_t be the energy per transmitted bit. The message is converted into a signal with energy $E_c = E_t \log_2 N$ that is transmitted using the corresponding chip (subchannel). This is equivalent to an N-FSK modulation, with frequencies $f_0 + n/\tau$, $n = 0, 1, ..., N-1$. If the j^{th} user is using the i^{th} chip, its activity factor c_n^j is equal to one; otherwise, it is equal to zero. Given the two following basic orthogonal forms with duration τ:

$$\begin{aligned} x_n(t) &= \sqrt{\frac{2E_c}{\tau}} \cos\left(2\pi \left[f_0 + \frac{n}{\tau}\right] t\right), \\ y_n(t) &= \sqrt{\frac{2E_c}{\tau}} \sin\left(2\pi \left[f_0 + \frac{n}{\tau}\right] t\right), \end{aligned} \tag{1}$$

the signal $s^j(t)$ transmitted by the j^{th} user is $s^j(t)$ and can be defined as follows:

$$s^j(t) = \sum_{n=0}^{N-1} c_n^j x_n(t). \tag{2}$$

The signal from each user suffers Rayleigh fading α_n^j and uniformly distributed phase rotation θ_n^j, all of them statistically independent. There is also a white Gaussian noise component $n(t)$ with density N_0. The received signal associated to the n^{th} chip can be written as:

$$r_n = \sum_{j=1}^{T} \left\{ c_n^j \alpha_n^j \left[cos(\theta_i^j) x_n(t) + sin(\theta_i^j) y_n(t) \right] \right\} + n(t). \tag{3}$$

Total energy per chip can be detected using a pair of matched filters, one for each basic orthogonal forms in 1, resulting in the values X_n and Y_n:

$$\begin{aligned} X_n &= E_c^{-1} \int_0^{\tau} r_n(t) x_n(t), \\ Y_n &= E_c^{-1} \int_0^{\tau} r_n(t) y_n(t). \end{aligned} \tag{4}$$

The output from the energy detector is $R_n = X_n^2 + Y_n^2$. If $c_n = \sum_{j=1}^{T} c_n^j$ is the total number of users that are transmitting in the n^{th} chip, R_n has an exponential distribution with parameter $c_n + d$, with $d = N_0/E_c$. Since the output R_n depends exclusively on c_n, the system can be seen as concatenation of a noiseless multiple access channel with N parallel noisy channels. The values of R_n are used by the MUD to provide information about the $T \times I$ information bits.

III. ITERATIVE DETECTOR AND DECODER

The receiver performs iterative multiuser detection and decoding. Information is exchanged between a common MUD and T parallel channel decoders, one for each user, as shown

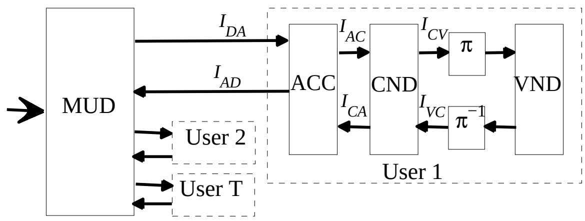

Fig. 1. Iterative receiver with details of decoder layers for the first user

in Fig. 1. Each decoder stops its processing when a valid codeword is found. The iterative process stops when all decoders have stopped or a fixed number of iterations has been reached (100 iterations in this case).

A. Multiuser detector

For an iterative system, a multiuser detector should be able to use *a priori* information about all input bits and the energy detectors' output to generate *a posteriori* extrinsic information about the bits. This could be done by applying the sum product algorithm over a factor graph that relates the bit probabilities to the received values. The statistical relationship between the set of bits and the received set of signals R_n can be done by a joint p.d.f $p(\mathbf{R}, \mathbf{b})$ or its extended version $p(\mathbf{R}, \mathbf{c}, \mathbf{c}^j, \mathbf{m}, \mathbf{b})$. The variables $\mathbf{R}$ and $\mathbf{c}$ are N-dimensional vectors containing all values of R_n and c_n respectively. The variable $\mathbf{c}^j$ is a $n \times j$ matrix containing the values of c_n^j. The variable $\mathbf{m}$ is a T dimensional vector containing the values of m^j. The variable $\mathbf{b}$ is a $K \times T$ matrix with the values of b_i^j. Variable indexes relate to matrix or vector indexes according to the dimensions involved.

The extended version $p(\mathbf{R}, \mathbf{c}, \mathbf{c}^j, \mathbf{m}, \mathbf{b})$ can be factored as:

$$\begin{aligned} p(\mathbf{R}, \mathbf{c}, \mathbf{c}^j, \mathbf{m}, \mathbf{b}) &= p(\mathbf{R}|\mathbf{c}) P(\mathbf{c}|\mathbf{c}^j) P(\mathbf{c}^j|\mathbf{m}) P(\mathbf{m}|\mathbf{b}) P(\mathbf{b}) \\ p(\mathbf{R}|\mathbf{c}) &= \prod_{n=0}^{N-1} p(R_n|c_n) \\ P(\mathbf{c}|\mathbf{c}^j) &= \prod_{n=0}^{N-1} P(c_n|c_n^1, c_n^2, ..., c_n^T) \\ P(\mathbf{c}^j|\mathbf{m}) &= \prod_{j=1}^{T} \prod_{n=0}^{N-1} P(c_n^j|m^j) \\ P(\mathbf{m}|\mathbf{b}) &= \prod_{j=1}^{T} P(m^j|b_0^j, b_1^j, ..., b_{K-1}^j) \end{aligned} \tag{5}$$

Using the Iverson bracket[6], the deterministic relationships between variables can be converted into conditional p.m.f.'s as follows:

$$\begin{aligned} P(m^j|b_0^j, b_1^j, ..., b_{K-1}^j) &= \left[m^j = \sum_{k=0}^{K-1} b_k^j 2^K \right] \\ P(c_n^j|m^j) &= [m^j = n][c_n^j = 1] + [m^j \neq n][c_n^j = 0] \\ P(c_n|c_n^1, c_n^2, ..., c_n^T) &= \left[c_i = \sum_{j=1}^{T} c_n^j \right] \end{aligned} \tag{6}$$

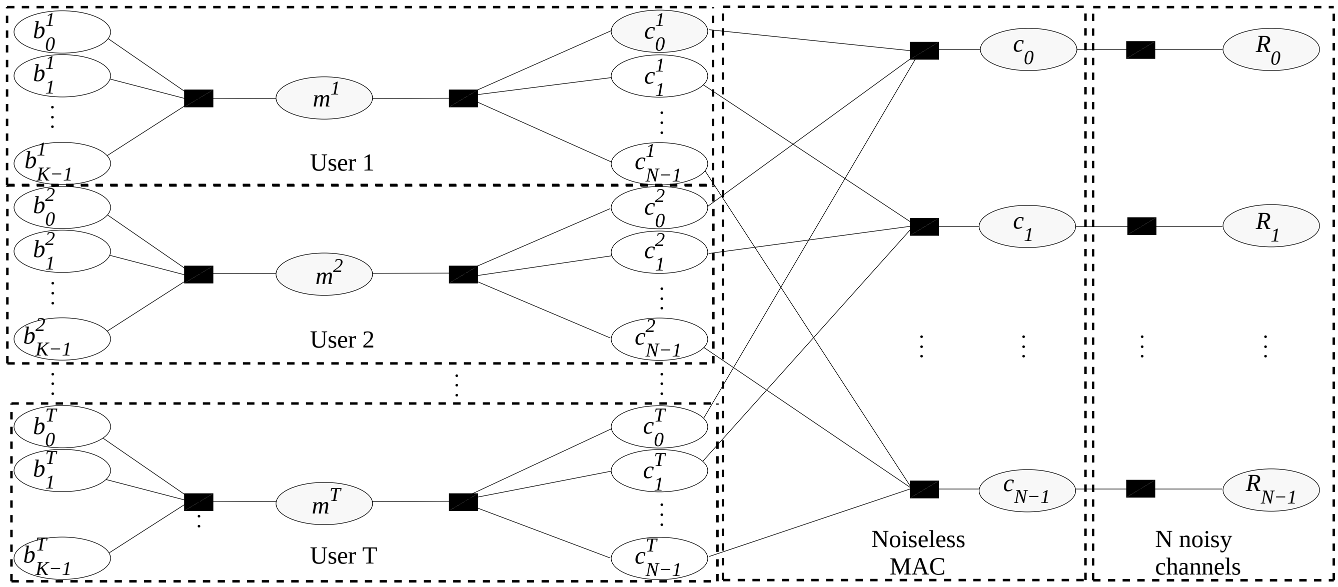

Fig. 2. Factor graph for the multiuser detector.

Representing the variables as circles and local functions as squares, the resulting factor graph is shown in Fig. 2. The graph can be decomposed in smaller graphs representing each user, the multiple access noiseless channel and the N parallel noisy channels, as indicated. The same graph can be used to other situations such as Pulse Position Modulation(PPM) with few adaptations.

B. EXIT analysis

The system can be analyzed using EXIT charts [5]. The analysis will be performed for the MUD connected with T parallel decoders interface.

The EXIT curve for the MUD can be obtained by simulation. Since there is interference between users, transmitted bits should be randomly generated for all users. From a set of random bits, values for R_n can be randomly generated using the equations of section II. Input messages containing *a priori* information about the transmitted bits should be feed to the multiuser detector at the corresponding nodes. These messages are generated using a Gaussian distribution with mean and variance relating to the amount of information to be provided as indicated by the $J(.)$ function and its inverse[5]. Since the graph has cycles, there is no natural stopping criteria. Results show that 5 internal iterations are sufficient to provide a stable EXIT curve for the detector, where an internal iteration happens when all nodes from left to right and back to left, according to Fig. 2, generate new messages.

The equivalent graph for a systematic RA code can be considered as a concatenation of three layers as seen in Fig. 1: Variable node decoder (VND), Check node decoder (CND) and Accumulator(ACC). EXIT charts for decoders are determined exclusively by the degree distributions $d_v(a)$ and $d_c(b)$ that indicate respectively the fraction of nodes from the VND and CND layers that have a and b connections with the other layer. Since all codes have by project the same degree distributions, all decoders have the same EXIT curve and can be treated as a single entity to be optimized.

In [7], EXIT charts for RA codes were optimized considering the interface between the VND and the CND layers. A more suitable approach to the problem studied in this paper is to combine the EXIT curves from each of these layers so that the decoder as a whole has a single EXIT curve. Let $I_{AB}(.)$ be the EXIT function that indicates how much information is transmitted from layer A to layer B, where A and B can assume the values of V (VND), C (CND), A (ACC) and D (MUD). To combine the curves of the VND and CND layers it is needed to find the point (I_a, I_b) such that $I_{VC}(I_a) = I_b$ and $I_{CV}(I_b, I_{AC}) = I_a$, that is, a stability point in the EXIT curves of these layers, given that the accumulator is providing I_{AC} of information. The amount of information to be returned to the accumulator is $I_{CA}(I_b, I_{AC})$, which is in fact only a function of I_{AC}. The same procedure can be done to combine this curve with the accumulator's EXIT curve to obtain $I_{AD}(I_{DA})$.

IV. CODE PROJECT

For given values of T, N and E_t/N_o, the MUD's EXIT curve can be determined and is invariable. The encoder can be optimized by fitting the decoders EXIT curve to the MUD's EXIT curve. Fitting is done by adjusting the parameters $d_v(.)$ and $d_c(.)$. Since, by project, all users have encoders with the same set of parameters and all decoders operate in parallel, it can be assumed that all decoders will provide the same amount of information about the transmitted bits. Thus, all decoders can be seen as a single decoder whose curve should be fitted to the MUD's EXIT curve.

Optimization can be done by maximizing the code rate, with the constraint that the decoder's EXIT curve remains below the MUD's EXIT curve, that is, $I_{AD}(I_{DA}(x)) \geq x, 0 \leq x \leq 1$. Since it does not seem possible to evaluate analytically this restriction for all values of x, the constraint can be simplified to be tested only for a finite number of values of x. The advantage of this method is that the function that relates the code rate to the parameters $d_v(.)$ and $d_c(.)$ is simple enough to allow calculation of Gradient and Hessian functions, improving the optimization speed.

To simplify the process, the values of a such that $d_v(a) > 0$ were limited to 3, 4 and 6. The values of b such that $d_c(b) > 0$ were limited to 1, 2, 3 and 12. Results for some combinations of N, T are shown in Tab. I, where $(E_t/N_0)_p = 5dB$ by project. A sample of the curve fitting is shown in Fig. 3.

Although all users have the same set of parameters, they do not share the same encoder implementation. To do so would make it impossible to determine which user transmitted which codeword, since it would result in the same set of codewords for more than one user. Let the total number of branches connecting the VND and CND layers be equal to ρ. The branches can be indexed in some order in each of these layers. The branch permutation function $\pi(a) = b$, $a, b = 1, 2, ..., \rho$, indicates the the a^{th} branch of the CND is connected to the b^{th} branch of the VND. By using different branch permutation functions, different encoders for the same set of parameters can be obtained.

TABLE I

OPTIMIZED RA CODE PARAMETERS

System		A	B	C	D
N		4	4	8	16
T		2	3	2	3
Rate		0.3711	0.2761	0.5156	0.5267
d_v	3	0	0.9992	0.9	0.9
	4	1	0	0.1	0.1
	6	0	0.0008	0	0
d_c	1	0.32	0.9868	0	0.55
	2	0	0	0.6	0.25
	3	0.68	0	0.3	0
	12	0	0.0132	0.1	0.2

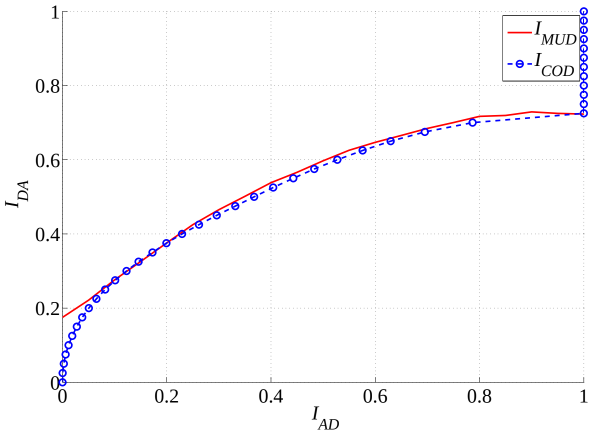

Fig. 3. EXIT curves fitting for system D.

V. RESULTS

Bit error rate (BER) simulation results for the codes presented in Tab. I are shown in Fig. 4. Enough bits were transmitted so as to assure that the values are within 2% of the correct value with 96% reliability. Since there is interference, the all zero codeword cannot be assumed. The permutation functions $\pi()$ were randomly obtained and exchanged every 10 transmission words. All codes have block length equal to 10^5 bits. For system A, BER is also shown for block lengths of 10^4 and 10^3 bits.

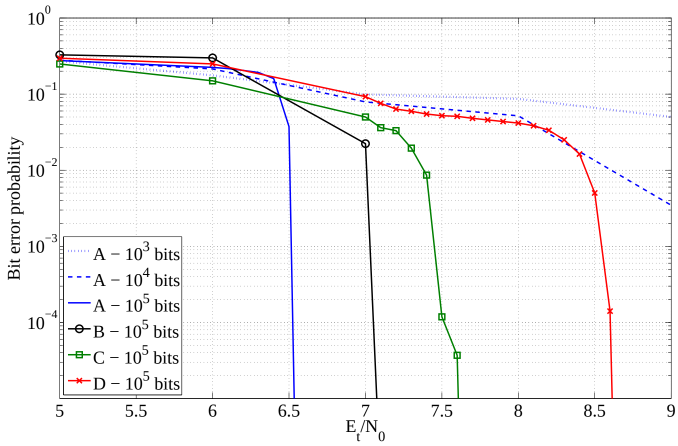

Fig. 4. Bit error probability for systems in table I

System rates are compared to channel sum capacity in Tab. II. The table shows the system rate; channel capacity at project $(E_t/N_0)_p = 5dB$; the value of $(E_t/N_0)_*$ such that channel capacity is equal to system rate; the operation value $(E_t/N_0)_o$ such that BER is lower than 10^{-5}. For comparison reasons, the value $(E_t/N_0)_s$ indicates how much should this ratio be to allow the same sum rate when single user detection is employed. The project distance can be seen as the difference $\Delta_p = (E_t/N_0)_p - (E_t/N_0)_*$. This value could be improved (reduced) by a better project, which could be obtained by relaxing the constrains imposed in the optimization process or by using another class of codes. Systems A and B work at less then $3dB$ from capacity. Systems B and D work with rates that are only possible due do multiuser detection.

TABLE II

COMPARISON OF SYSTEM RATE AND CHANNEL CAPACITY

Code	Rate	Capacity	$\left(\frac{E_t}{N_0}\right)_*$	$\left(\frac{E_t}{N_0}\right)_o$	$\left(\frac{E_t}{N_0}\right)_s$
A	0,3711	0,4198	3,9	6,7	6,5
B	0,2761	0,2958	4,4	7,1	9
C	0,5156	0,5388	4,5	7,7	7,1
D	0,5267	0,5352	4,8	8,6	9,4

To the authors' knowledge these are the first results of capacity approaching codes for a multiple access channel with frequency selective fast fading and non-coherent detection. Even under this conditions, it was possible to achieve rates close to channel capacity without the need of a hopping access code. Results were presented in [8] and [9] for a binary input multiple access channel with only Gaussian noise and coherent detection.

VI. Conclusion

In this work an efficient system for transmission in a frequency selective fast fading channel was presented. The system proposed is a simplification of a FFH-CDMA system because the hopping access code is eliminated. Regular channel coding is sufficient to allow multiple access with rates close to channel capacity. Given that RA codes were to be employed, the choice of which RA code to use was done using EXIT charts. A multiuser detector, suitable for other situations such as PPM, was modeled and analyzed using a factor graph. The channel model, combined with noncoherent detection, makes this system suitable for situations where the channel cannot be estimated.

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