Lecture 6 - The Basic Feedback Loop

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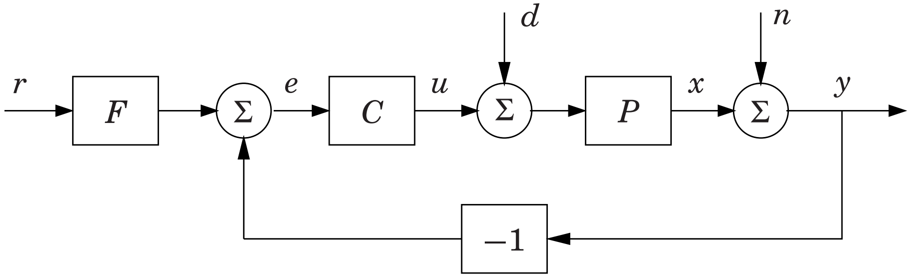
1. Introduction
2. Controllers with two degrees of freedom
3. The Gangs of Four and Six
4. The sensitivity functions
5. Summary

Theme: Understanding the basic feedback loop. Systems with two degrees of freedom. The gangs of four and six. Sensitivity functions

1. Introduction

- A nice collection of tools have been developed
- We have looked at a few examples
- We will now investigate a typical control problem
- A basis for control system design
- How to judge a control system
- New concepts and insight
  - Sensitivity functions
  - Deeper understanding of feedback
- A basis for a serious look at the design problem
- How to capture a complex reality in tractable mathematics

A Basic Control System



Ingredients:

- Controller: feedback C, feedforward F
- Load disturbance d: Drives the system from desired state
- Measurement noise n: Corrupts information about x
- Process variable x should follow reference r

Key Issues

Find a controller that

A: Reduces effects of load disturbances

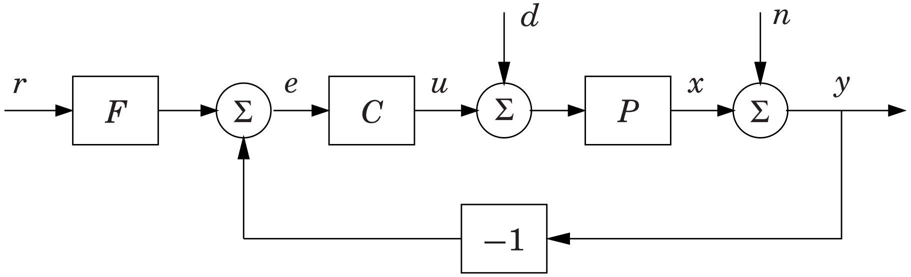
B: Does not inject to much measurement noise into the system

C: Makes the closed loop insensitive to variations in the process

D: Makes output follow command signals

Convenient to use a controller with two degrees of freedom, i.e. separate signal transmission from y to u and from r to u. This gives a complete separation of the problem: Use feedback to deal with A, B, and C. Use feedforward to deal with D!

2. System with Two Degrees of Freedom



The controller has two degrees of freedom (2DOF) because the transfer function from reference r to control u is different from the transfer function from y to u.

We have already encountered this in PI control

$$u(t) = k(br(t) - y(t)) + \int_0^t (r(\tau) - y(\tau))d\tau$$

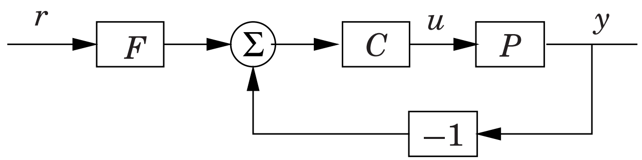
Designing System with Two Degrees of Freedom

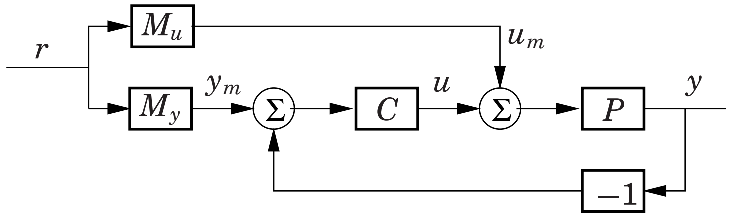
Design procedure:

- Design the feedback C to achieve
  - Small sensitivity to load disturbances d
  - Low injection of measurement noise n
  - High robustness to process variations
- Then design the feedforward F to achieve desired response to command signals r

For many problems in process control the load disturbance response is much more important than the set point response. The set point response is more important in motion control. Few textbooks and papers show more than set point responses.

Many Versions of 2DOF

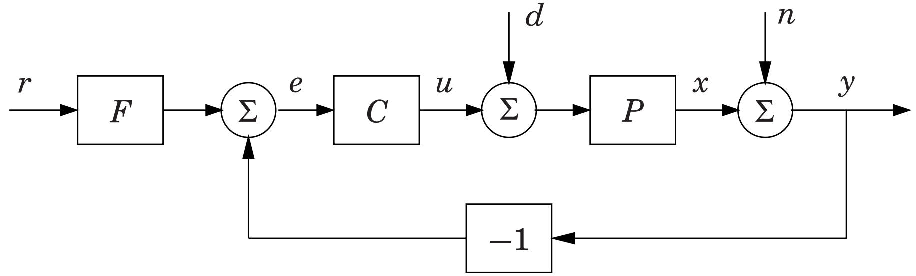




For linear systems all 2DOF configurations have the same properties. For the systems above we have

$$CF = M_u + CM_y$$

3. Relations between signals



$$X = \frac{P}{1+PC}D - \frac{PC}{1+PC}N + \frac{PCF}{1+PC}R$$
$$Y = \frac{P}{1+PC}D + \frac{1}{1+PC}N + \frac{PCF}{1+PC}R$$
$$U = -\frac{PC}{1+PC}D - \frac{C}{1+PC}N + \frac{CF}{1+PC}R$$

Some Observations

- A system based on error feedback is characterized by *four* transfer functions (The Gang of Four)
- The system with a controller having two degrees of freedom is characterized by *six* transfer function (The Gang of Six)
- To fully understand a system it is necessary to look at **all** transfer functions
- It may be strongly misleading to only show properties of a few systems for example the response of the output to command signals. This is a common error in the literature.
- The properties of the different transfer functions can be illustrated by their transient or frequency responses.

A Possible Choice

Six transfer functions are required to show the properties of a basic feedback loop. Four characterize the response to load disturbances and measurement noise.

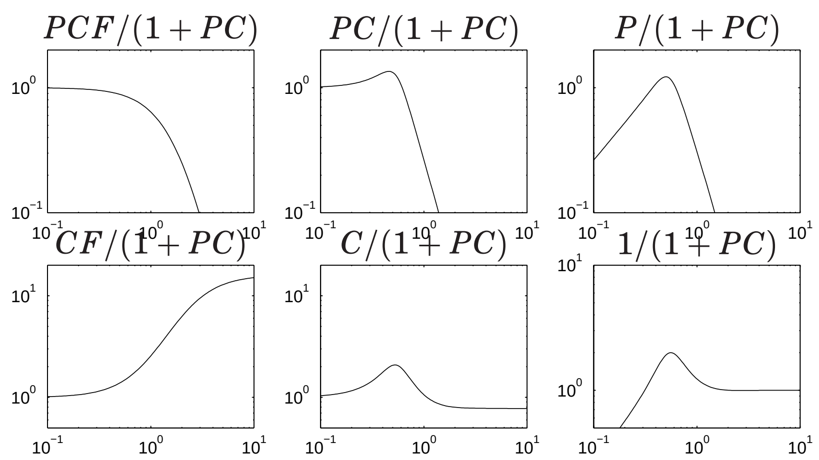
$$\begin{array}{cc} \dfrac{PC}{1+PC} & \dfrac{P}{1+PC} \\ \dfrac{C}{1+PC} & \dfrac{1}{1+PC} \end{array}$$

Two more are required to describe the response to set point changes.

$$\frac{PCF}{1+PC} \qquad \frac{CF}{1+PC}$$

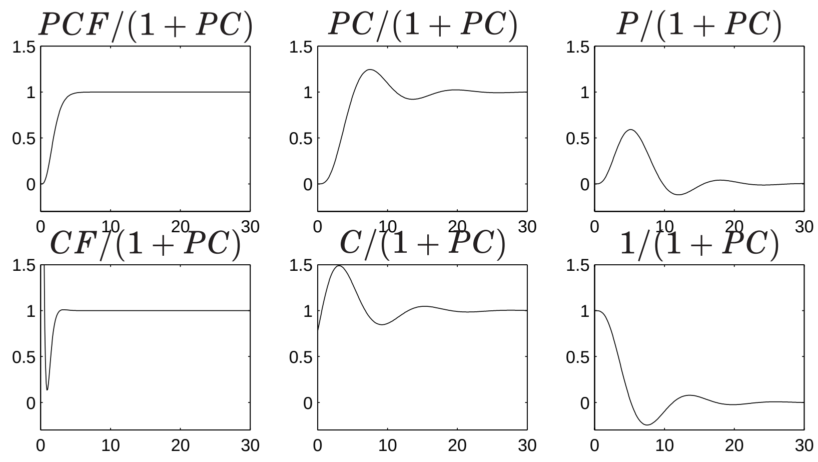
Amplitude Curves of Frequency Responses

PI control $k = 0.775$, $T_i = 2.05$ of $P(s) = (s+1)^{-4}$ with $M(s) = (0.5s+1)^{-4}$



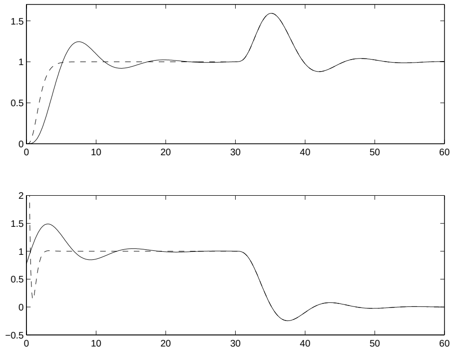
Step Responses

PI control $k = 0.775$, $T_i = 2.05$ of $P(s) = (s+1)^{-4}$ with $M(s) = (0.5s+1)^{-4}$



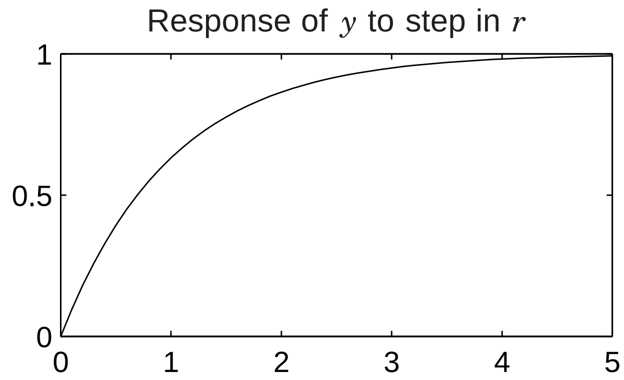
An Alternative

Show the responses in the output and the control signal to a step change in the reference signal for system with pure error feedback and with feedforward. Keep the reference signal constant and make a unit step in the process input. Show the response of the output and the control signal.

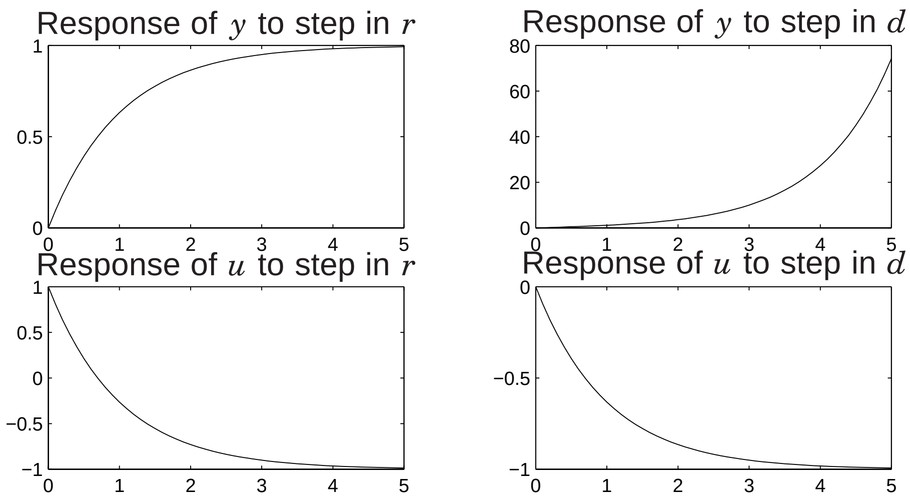


A Warning!

Please remember to always look at **all** responses when you are dealing with control systems. The step response below looks fine but ...



Four Responses



What is going on?

The System

Process $P(s) = \dfrac{1}{s-1}$

Controller $C(s) = \dfrac{s-1}{s}$

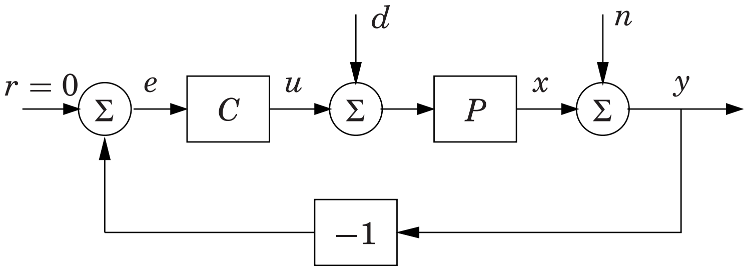
Response of y to reference r

$$\frac{Y(s)}{R(s)} = \frac{PC}{1+PC} = \frac{1}{s+1}$$

Response of y to step in disturbance d

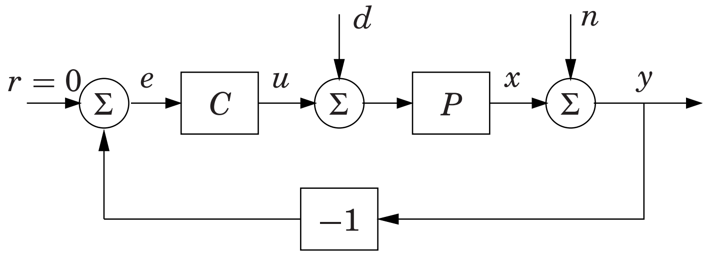
$$\frac{Y(s)}{D(s)} = \frac{P}{1+PC} = \frac{s}{s^2-1} = \frac{s}{(s+1)(s-1)}$$

Focus on Feedback



- Neglect following of reference signals (the feedforward problem).
- Focus on on the feedback problem
  - Load disturbances
  - Measurement noise
  - Model uncertainty

The Feedback Problem

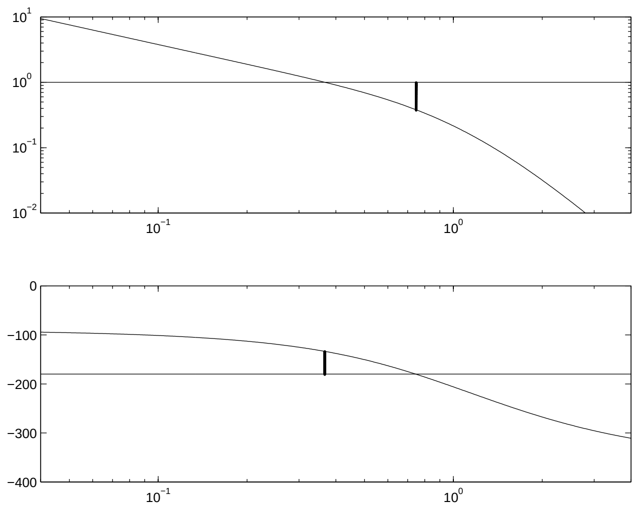


The signals have the following relations. Notice that there are only four transfer functions - The Gang of Four.

$$
\begin{aligned}
X &= \frac{P}{1+PC}D - \frac{PC}{1+PC}N \\
Y &= \frac{P}{1+PC}D + \frac{1}{1+PC}N \\
U &= -\frac{PC}{1+PC}D - \frac{C}{1+PC}N
\end{aligned}
$$

The Loop Transfer Function $L(s) = P(s)C(s)$

Tells a lot about the system.



4. The Sensitivity Functions

The transfer functions

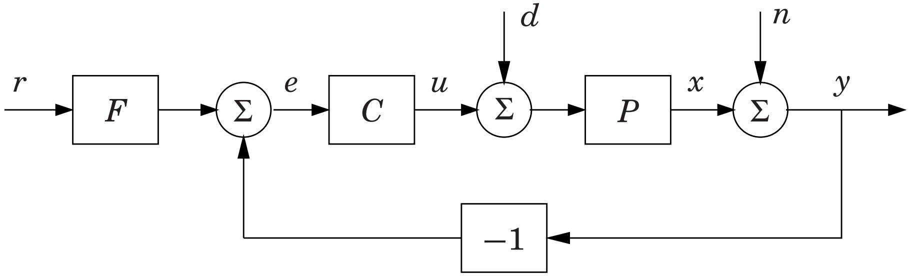
- Sensitivity function $S = \dfrac{1}{1+PC} = \dfrac{1}{1+L}$
- Complementary sensitivity function $T = \dfrac{PC}{1+PC} = \dfrac{L}{1+L}$

are called sensitivity functions. They have interesting properties and useful physical interpretations. We have

- The functions S and T only depend on the loop transfer function L
- $S + T = 1$
- Typically $S(0)$ small and $S(\infty) = 1$ and consequently $T(0) = 1$ and $T(\infty)$ small

Quiz

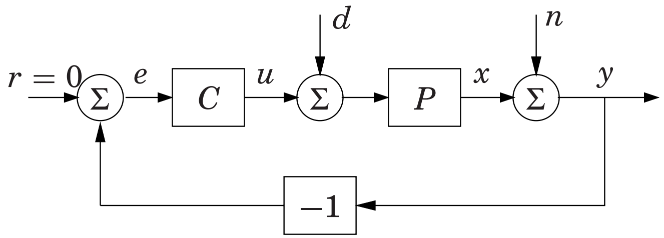
Look at the block diagram



Find all relations where the signal transmissions are equal to either the sensitivity function or the complementary sensitivity function

The Audience is Thinking ...

Disturbance Reduction



Output without control $Y = Y_{ol}(s) = N(s) + P(s)D(s)$

Output with feedback control

$$Y_{cl} = \frac{1}{1+PC}(N + PL) = \frac{1}{1+PC}Y_{ol} = SY_{ol} = SY_{ol}$$

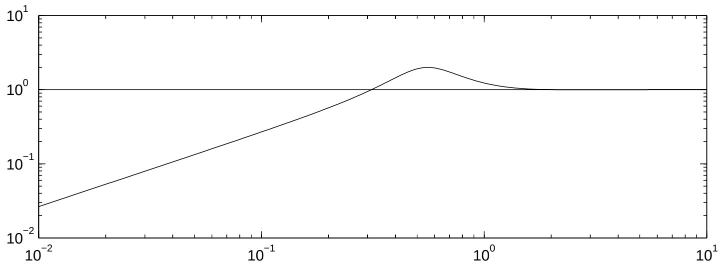
Disturbances with frequencies such that $|S(i\omega)| < 1$ are reduced by feedback, disturbances with frequencies such that $|S(i\omega)| > 1$ are amplified by feedback.

Assessment of Disturbance Reduction

We have

$$\frac{Y_{cl}(s)}{Y_{cl}(s)} = S(s) = \frac{1}{1 + P(s)C(s)}$$

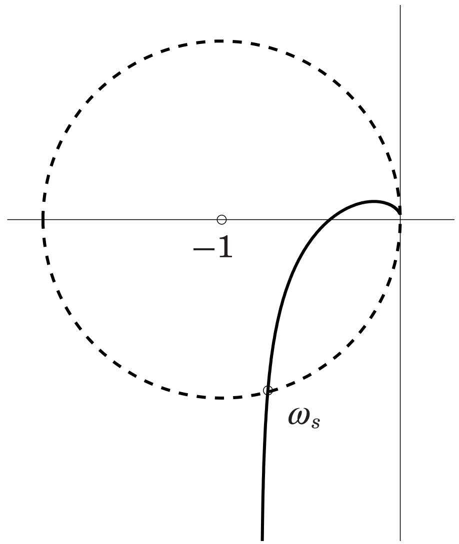
Feedback attenuates disturbances of frequencies ω such that $|S(i\omega)| < 1$. It amplifies disturbances of frequencies such that $|S(i\omega)| > 1$



Assessment of Disturbance Reduction

$$\frac{Y_{cl}}{Y_{ol}} = \frac{1}{1+PC} = S$$

Geometric interpretation: Disturbances with frequencies inside the circle are amplified by feedback. Disturbances with frequencies outside are reduced. Disturbances with frequencies less than ω_s are reduced by feedback.



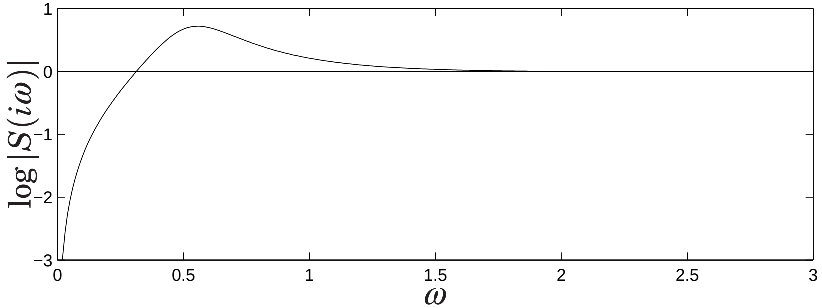
Properties of the Sensitivity function

- Can the sensitivity be small for all frequencies?
  - No we have $S(\infty) = 1$!
- Can we get $|S(i\omega)| \le 1$?
  - If the Nyquist curve of $L = PC$ is in the first and third quadrant! Passive systems!
- Bodes integral, p_k RHP poles of $L(s)$

$$\int_0^\infty \log |S(i\omega)| d\omega = \int_0^\infty \log \frac{1}{|1 + L(i\omega)|} d\omega$$
$$= \pi \sum \mathrm{Re}\ p_k - \frac{\pi}{2} \lim_{s\to\infty} sL(s)$$

- The "water-bed effect". Push the curve down at one frequency and it pops up at another!

The Water Bed Effect



$$\int_0^\infty \log |S(i\omega)| d\omega = \pi \sum \mathrm{Re}\ p_k - \frac{\pi}{2} \lim_{s\to\infty} sL(s)$$

The sensitivity can be decreased at one frequency at the cost of increase at another frequency.

Robustness

Effect of small process changes on $T = PC/(1 + PC)$

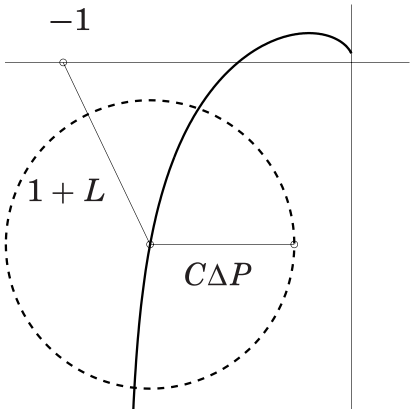
$$\frac{dT}{T} = \frac{dP}{P} - \frac{CdP}{1 + PC} = \frac{1}{1 + PC}\frac{dP}{P} = S\frac{dP}{P}$$

How much can the process be changes without making the system unstable?

$$|C\Delta P| < |1 + PC|$$

or

$$\frac{|\Delta P|}{|P|} < \frac{1}{|T|}$$

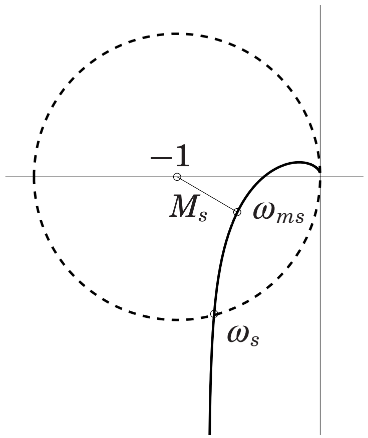


Maximum Sensitivity

The number

$$M_s = \max |S(i\omega)|$$

is a measure of robustness. The number $1/M_s$ is the smallest distance from the Nyquist curve to the critical point -1.

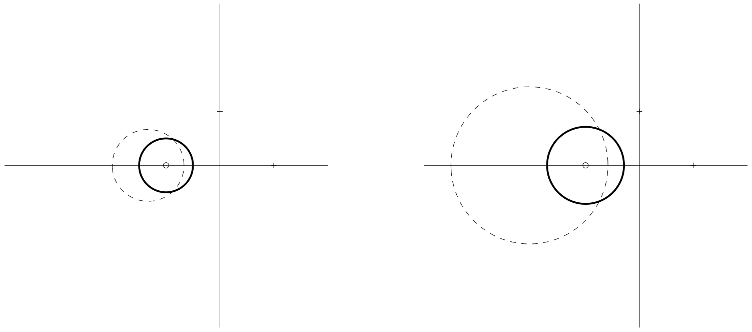


Reasonable values are between 1.2 and 2.

Maximum Sensitivities

Requirement on maximum sensitivities give constraints that tell that the Nyquist curve should avoid certain circles

$M_s = M_t = 2$ $\qquad$ $M_s = M_t = 1.4$



Summary of the Sensitivity Functions

$$S = \frac{1}{1+L}, \quad T = \frac{L}{1+L}, \quad M_s = \max|S(i\omega)|, \quad M_t = \max|T(i\omega)|$$

The value $1/M_s$ is the shortest distance from the Nyquist curve of the loop transfer function $L(i\omega)$ to the critical point -1.

$$S = \frac{\partial \log T}{\partial \log P} = \frac{Y_{cl}(s)}{Y_{ol}(s)}$$

How much can the process be changed without making the system unstable?

$$\frac{|\Delta P|}{|P|} < \frac{1}{|T|}$$

Bode's integral the water bed effect.

$$\int_0^\infty \log|S(i\omega)| d\omega = \pi \sum \mathrm{Re}\; p_k - \frac{\pi}{2} \lim_{s\to\infty} sL(s)$$

5. Summary

- Systems with two degrees of freedom allow a complete separation of responses to reference signals and disturbances.
- Design for disturbances and robustness first, then choose feedforward F to give desired response to reference signals
- A system with error feedback is characterized by four transfer functions (Gang of Four)
- The basic feedback loop with two degrees of freedom is characterized by six transfer functions (Gang of Six)
- The effect of feedback on disturbances is given by

$$Y_{closedloop}(s) = S(s) Y_{openloop}(s)$$

The Gangs of Four and Six

Response of y to load disturbance d is characterized by

$$\frac{P}{1+PC}$$

Response of u to measurement noise n is characterized by

$$\frac{C}{1+PC}$$

Robustness to process variations is characterized by

$$S = \frac{1}{1+PC}, \quad T = \frac{PC}{1+PC}$$

Responses of y and u to reference signal r is characterized by

$$\frac{PCF}{1+PC}, \quad \frac{CF}{1+PC}$$