Lecture 9 - Review and Assessment

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1. Introduction
2. Design of Simple Controllers
3. The Basic Feedback Loop
4. Specifications
5. Assessment
6. Summary

Theme: Review of what we learned so far. Assessment of a control systems is important, particularly if you are a customer of control.

1. Introduction

- Mid course summary
  - What have we achieved so far
  - What remains
- Standard models
  - Block diagrams
  - Differential equations
  - Transfer functions
- Design of simple controllers
- Insight into feedback and feedforward
- The basic feedback loop
- Fundamental limitations

Block Diagrams

The block diagram gives an overview of a system. To draw a block diagram:

- Understand how the system works.
- Identify the major components and the relevant signals.
- Key questions:

  Where is the essential dynamics?

  What are appropriate abstractions?

  Think about physics and storage of mass, energy and momentum
- Describe the dynamics of the blocks in terms of standard models.

Standard Model 1 - Differential Equations

A standard model for linear time invariant system

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1}y}{dt^{n-1}} + \ldots + a_n y = b_1 \frac{d^{n-1}u}{dt^{n-1}} + \ldots + b_n u$$

is characterized by two polynomials

$$A(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_{n-1}s + a_n$$
$$B(s) = b_1 s^{n-1} + b_2 s^{n-2} + \ldots + b_{n-1}s + b_n$$

- The roots of $A(s)$ are called poles of the system.
- The roots of $B(s)$ are called zeros of the system.
- The transfer function of the system is $G(s) = \dfrac{B(s)}{A(s)}$

Standard Model 1 ...

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \ldots + a_n y = b_1 \frac{d^{n-1} u}{dt^{n-1}} + \ldots + b_n u$$

has the solution

$$y(t) = \sum_{k=1}^{n} C_{k-1}(t) e^{\alpha_k t} + \int_0^t g(t-\tau) u(\tau) d\tau$$

where $\{\alpha_k\}$ are roots of the characteristic equation A, $C_k(t)$ polynomials and g is the *impulse response*, which has the form

$$g(t) = \sum_{k=1}^{n} \bar{C}_{k-1}(t) e^{\alpha_k t}$$

Notice appearance of α_k again!

Experimental determination of step and impulse responses.

Standard Model 2 - Transfer Functions

The transfer function can also be defined as

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\mathcal{L}y}{\mathcal{L}u}$$

$$\mathcal{L}f = F(s) = \int_0^\infty e^{-st} f(t) dt$$

where *the Laplace transforms are calculated under the assumption that all initial values are zero.*

Transfer functions and Laplace transforms are ideal to deal with block diagram. A block is simply characterized by

$$Y(s) = G(s)U(s)$$

Signals and and systems have the same representations.

Frequency Responses

The complex number $G(i\omega)$ tells how a sinusoid propagates through the system *in steady state*. If the input is $u(t) = \sin \omega t$, then the output is

$$y(t) = |G(i\omega)| \sin\left(\omega t + \arg G(i\omega)\right)$$

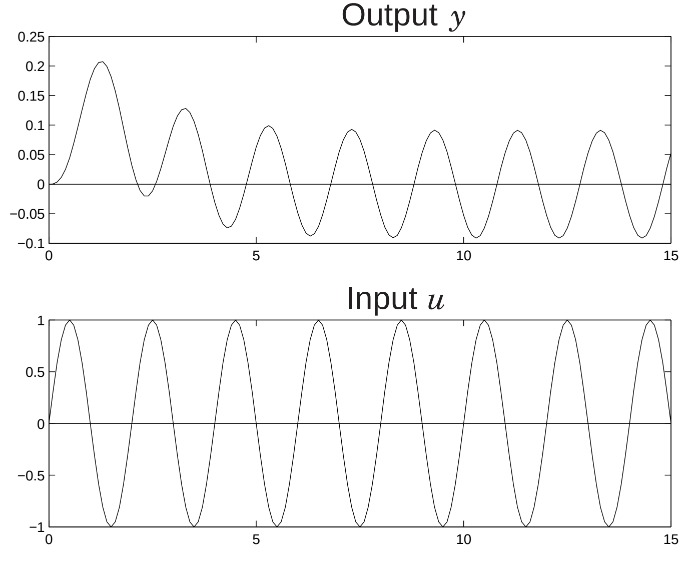
The number $|G(i\omega)|$ is called gain ratio or simply gain and the number $\arg G(i\omega)$ is called phase of the transfer function.

Graphical representations

- Nyquist plot
- Bode plot

Nyquist's stability criterion, stability margins

Notice Steady State Responses



3. Design of Simple Controllers

- Draw block diagram
- Obtain simple process model
- Select a controller of sufficient flexibility (complexity)
- Derive the closed loop characteristic equation
- Pick controller parameters to give desired characteristic polynomial (pole placement). Typical prototypes:

  $$s^2 + 2\zeta\omega_0 s + \omega_0^2$$
  $$(s^2 + 2\zeta\omega_0 s + \omega_0^2)(s + \omega_0)$$

- Behavior of second and third order systems
- The effect of poles and zeros
- Some zeros are due to the controller, they can be modified or eliminated if the controller has two degrees of freedom.

PI Control of First Order Systems

Process transfer function

$$P(s) = \frac{b}{s+a}$$

Controller with 2DOF

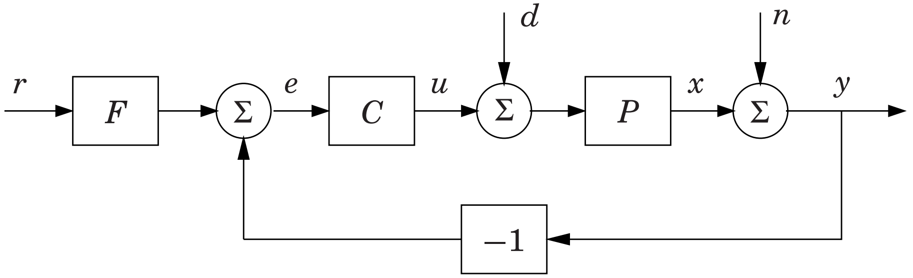
$$U(s) = -kY(s) + \frac{k_i}{s}(R(s) - Y(s))$$

Closed loop transfer function from reference r to output y

$$\frac{Y(s)}{R(s)} = \frac{bk_i}{s^2 + (a+bk)s + bk_i} = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

Closed loop system of second order, PI controller has two parameters, parameters ω_0 and ζ can be chosen freely, but ...

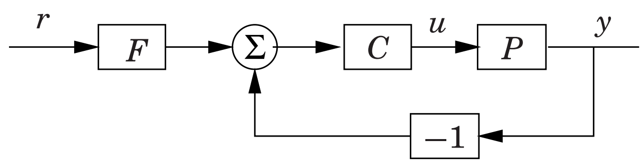
4. The Basic Feedback Loop

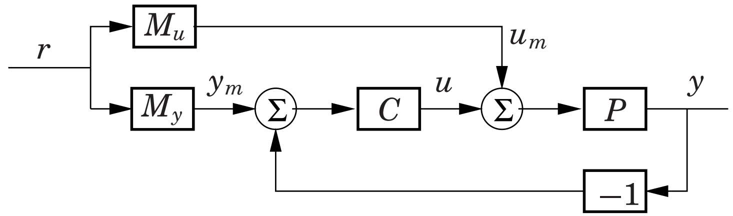


Ingredients:

- Controller: feedback C, feedforward F
- Load disturbance d: Drives the system from desired state
- Measurement noise n: Corrupts information about x
- Process variable x should follow reference r

Many Versions of 2DOF





For linear systems all 2DOF configurations have the same properties. For the systems above we have

$$CF = M_u + CM_y$$

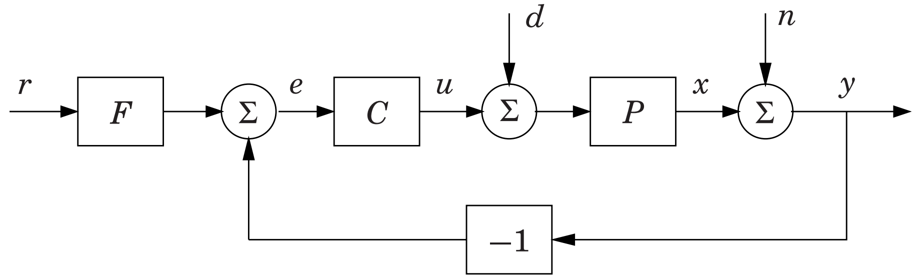
Designing System with Two Degrees of Freedom

Design procedure:

- Design the feedback C to achieve
  - Small sensitivity to load disturbances d
  - Low injection of measurement noise n
  - High robustness to process variations
- Then design the feedforward F to achieve desired response to command signals r. System inversion.

For many problems in process control the load disturbance response is much more important than the set point response. The set point response is more important in motion control. Few textbooks and papers show more than set point responses.

The Gang of Six



$$X = \frac{P}{1+PC}D - \frac{PC}{1+PC}N + \frac{PCF}{1+PC}R$$
$$Y = \frac{P}{1+PC}D + \frac{1}{1+PC}N + \frac{PCF}{1+PC}R$$
$$U = -\frac{PC}{1+PC}D - \frac{C}{1+PC}N + \frac{CF}{1+PC}R$$

Sensitivity Functions

$$S = \frac{1}{1+L}, \quad T = \frac{L}{1+L}, \quad M_s = \max|S(i\omega)|, \quad M_t = \max|T(i\omega)|$$

The value $1/M_s$ is the shortest distance from the Nyquist curve of the loop transfer function $L(i\omega)$ to the critical point -1.

$$S = \frac{\partial \log T}{\partial \log P} = \frac{Y_{cl}(s)}{Y_{ol}(s)}$$

How much can the process be changed without making the system unstable?

$$\frac{|\Delta P|}{|P|} < \frac{1}{|T|}$$

Bode's integral the water bed effect.

$$\int_0^\infty \log|S(i\omega)|d\omega = \pi \sum \mathrm{Re}\ p_k - \frac{\pi}{2}\lim_{s\to\infty} sL(s)$$

Rules of Thumb for Limitations - Part 1

- A RHP zero z

$$\frac{\omega_{gc}}{z} \le \begin{cases} 0.5 & \text{for } M_s,\ M_t < 2 \\ 0.2 & \text{for } M_s,\ M_t < 1.4. \end{cases}$$

- A time delay T

$$\omega_{gc}T \le \begin{cases} 0.7 & \text{for } M_s,\ M_t < 2 \\ 0.37 & \text{for } M_s,\ M_t < 1.4. \end{cases}$$

- A RHP pole p

$$\frac{\omega_{gc}}{p} \ge \begin{cases} 2 & \text{for } M_s,\ M_t < 2 \\ 5 & \text{for } M_s,\ M_t < 1.4. \end{cases}$$

Rules of Thumb for Limitations - Part 2

- A RHP pole-zero pair with $z > p$

$$\frac{z}{p} \geq \begin{cases} 6.5 & \text{for } M_s,\ M_t < 2 \\ 14.4 & \text{for } M_s,\ M_t < 1.4. \end{cases}$$

- A RHP pole-zero pair with $z < p$

$$\frac{p}{z} \geq \begin{cases} 6.5 & \text{for } M_s,\ M_t < 2 \\ 14.4 & \text{for } M_s,\ M_t < 1.4 \end{cases}$$

- A RHP pole p and a time delay T

$$pT \leq \begin{cases} 0.16 & \text{for } M_s,\ M_t < 2 \\ 0.05 & \text{for } M_s,\ M_t < 1.4. \end{cases}$$

5. Assessment of a Control System

- If you are designing control system it is important to assess their properties
- If you are a customer to control system designers is is useful to assess their properties
- If you are using control systems it is important to assess their performance
- You must consider all properties not just the response to reference signals

Examples

Previously we have seen that it was straight forward to use pole placement to design a controller for a given process. We use our new insight into the properties of the basic feedback loop and specifications to make an assessment of the design. Two simple examples are used as illustration.

- PI Control of a first order system
- PI Control of a second order system

Some interesting questions

- How should the closed loop poles be chosen?
- Are there any restrictions?

PI Control of a First Order System

Process model: $P = \dfrac{b}{s+a}$

Controller with 2 DOF: $U = -kY + \dfrac{k_i}{s}(R - Y)$

Loop transfer function: $L = \dfrac{b(ks + k_i)}{s(s+a)}$

Closed loop characteristic equation: $s^2 + s(a + bk) + bk_i = 0$

Choose $k = (2\zeta\omega_0 - 1)/b$ and $k_i = \omega_0^2/b$ to give

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

Are there any restrictions on ζ and ω_0?

The Gang of Six

Response to reference signal

$$Y(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} R(s), \quad U(s) = \frac{\omega_0^2(s+a)}{b(s^2 + 2\zeta\omega_0 s + \omega_0^2)} R(s$$

Response to load disturbance

$$Y(s) = \frac{bs}{s^2 + 2\zeta\omega_0 s + \omega_0^2} D(s)$$

Response to measurement noise

$$U(s) = -\frac{(s+a)(ks+k_i)}{s^2 + 2\zeta\omega_0 s + \omega_0^2} N(s) = -\frac{(s+a)((2\zeta\omega_0 - a)s + \omega_0^2)}{s^2 + 2\zeta\omega_0 s + \omega_0^2} N(s)$$

Sensitivity functions

$$S(s) = \frac{s(s+a)}{s^2 + 2\zeta\omega_0 s + \omega_0^2}, \quad T(s) = \frac{((2\zeta\omega_0 - a)s + \omega_0^2)}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

The Sensitivity Function

Sketch the gain curve of the Bode plot for the sensitivity function

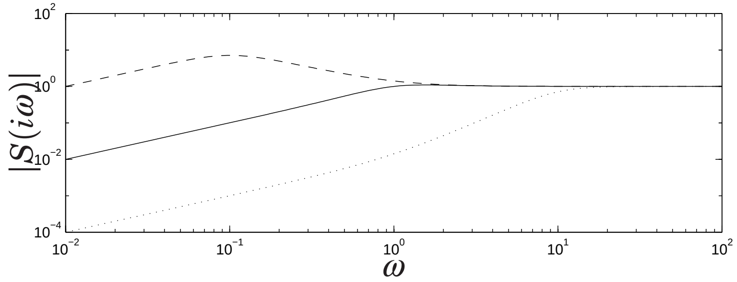
$$S(s) = \frac{s(s+a)}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

for $a = 1$, $\zeta = 0.5$ and $\omega_0 = 0.1$

The Audience is Thinking ...

The Sensitivity Function

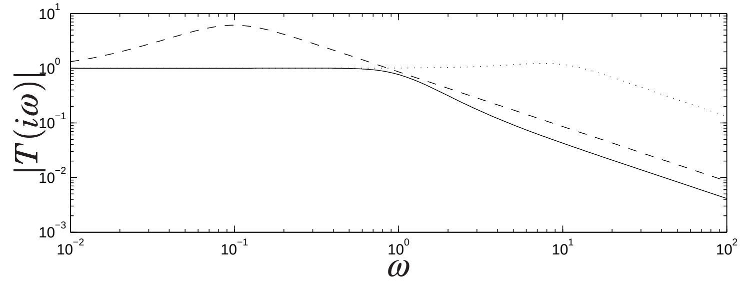
Gain curve of Bode plot for $\zeta = 0.7$, $a = 1$ and $\omega_0/a = 0.1$ (dashed), 1 (solid) and 10 (dotted)



Choosing ω_0 too small compared to a gives a system with large M_s large disturbance amplification. Notice this occurs even if relative damping ζ is well chosen!

The Complementary Sensitivity Function

Gain curve of Bode plot for $\zeta = 0.7$, $a = 1$ and $\omega_0/a = 0.1$ (dashed), 1 (full) and 10 (dotted)



Choosing ω_0 too small in comparison with a gives a system with poor robustness. Notice this occurs even if relative damping ζ is well chosen!

Conclusion

Process model: $P(s) = \frac{b}{s+a}$ Design a PI controller to give a closed loop system with the characteristic polynomial

$$s^2 + 2\zeta\omega_0 s + \omega_0^2$$

Controller parameters: $k = \frac{2\zeta\omega_0 - a}{b}, \quad k_i = \frac{\omega_0^2}{b}$

To obtain reasonable robustness the parameter ω_0 cannot be too small, ($\omega_0 \geq 0.5844a$ gives sensitivities less than 2.

Too high values of ω_0 give systems that are too sensitive to measurement noise $k = 2\zeta\omega_0$! Model uncertainty also restricts how large ω_0 can be!

Even if we can place the poles arbitrarily the process imposes limitations!

Control of a Thermal Process

You are in charge of a project concerning temperature control for a wafer production. A simple process model is

$$P(s) = \frac{1}{(s+1)(s+0.02)}$$

An engineer that you supervised has proposed

$$U = -Y + \frac{1}{50s}(R - Y)$$

He said that the response to a step in the reference signal originally had large overshoot which was eliminated by not applying proportional action to the reference. He showed you ice step responses to changes in the reference signal. He also showed that the maximal values of the sensitivity functions are reasonable. Has he given you a good controller?

The Gang of Six

Response to reference signal

$$Y(s) = \frac{1}{s^2+s+1}R(s), \quad U(s) = \frac{(s+1)(s+0.02)}{s^2+s+1}R(s$$

Response to load disturbance

$$Y(s) = \frac{s}{(s+0.02)(s^2+s+1)}D(s) \quad U(s) = -\frac{s+0.02}{s^2+s+1}D(s)$$

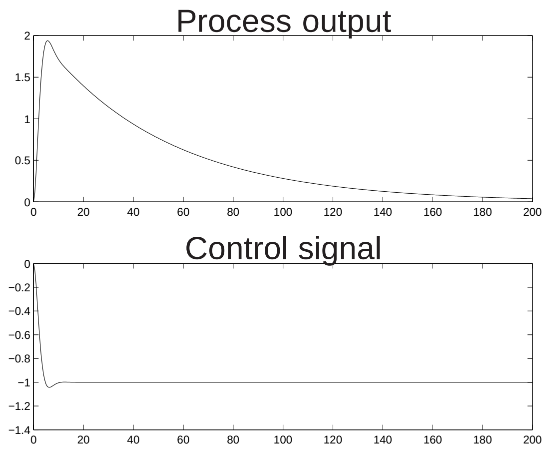
Response to measurement noise

$$U(s) = -\frac{(s+0.02)(s+1)}{s^2+s+1}N(s)$$

Sensitivity functions

$$S(s) = \frac{s(s+1)}{s^2+s+1}, \quad T(s) = \frac{1}{s^2+s+1}$$

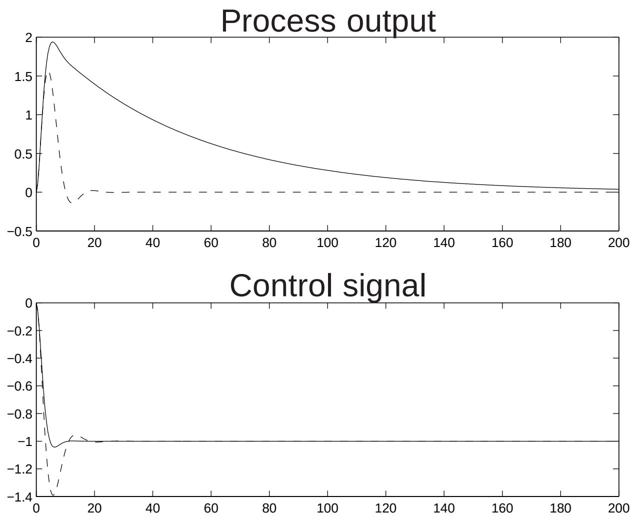
Response to Step in Load Disturbance



Very slow recover of load disturbance! Why does the control signal not eliminate it?

Drastically Improved Control

Changing T_i from 50 to 4 gives a drastic improvement



6. Summary

- Summarizing what we learned so far
- Insight into feedback and feedforward
- A good understanding of the basic feedback loop
- Important to look at all six transfer functions ("Gang of Six")
- Assessment gives insight into the design problem and guidance for choice of parameters
- What remains to do
  - How to obtain the models?
  - How to deal with many inputs and outputs?
  - Polynomials are bad numerically for high order
  - More efficient computations