Lecture 7 - Feedforward Design

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1. Introduction
2. Attenuation of measured disturbances
3. System inverses
4. Improved response to reference signals
5. Summary

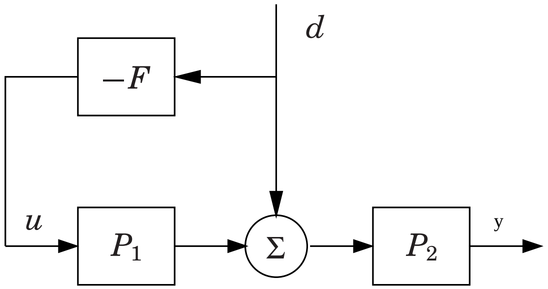
Theme: A simple and useful idea. System inverses.

1. Introduction

Feedforward is a useful complement to feedback. Some of its properties are:

+ Reduce effects of disturbances that can be measured
+ Improve response to reference signals
+ No risk for instability
- Design of feedforward is simple but it requires good models
+ Beneficial to combine with feedback

2. Attenuation of Measured Disturbance

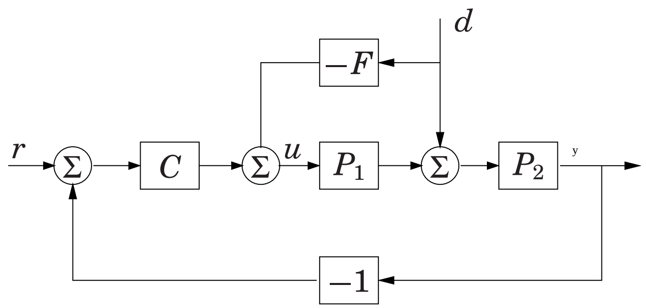


Transfer function from disturbance to process output with pure feedforward

$$\frac{Y(s)}{D(s)} = P_2(1 - P_1 F)$$

Disturbance is eliminated if $F = -P_1^{-1}$

Combined Feedback and Feedforward



Transfer function from disturbance to process output with combined feedback and feedforward

$$G_d = \frac{Y(s)}{D(s)} = \frac{P_2(1 - P_1 F)}{1 + PC} = P_2(1 - P_1 F)S$$

Notice combined effect of feedback and feedforward.

Sensitivity to Modeling Errors

Pure feedforward

$$G_d = P_2(1 - P_1F), \quad \frac{dG_s}{G_s} = \frac{dF}{F}$$

Combined feedback and feedforward

$$G_d = P_2(1 - P_1F)S, \quad \frac{dG_s}{G_s} = S\frac{dF}{F}$$

The Ideal Feedforward Compensator

Transfer function from disturbance to process output

$$\frac{Y(s)}{D(s)} = \frac{P_2(1 - P_1F)}{1 + PC} = P_2(1 - P_1F)S$$

What is a good feedforward compensator?

The choice

$$F = P_1^{-1} = \frac{P_{yd}}{P_{yu}}.$$

gives perfect compensation.

Design of feedforward is essentially a system inversion!

3. System Inverses

It is formally easy to invert transfer functions, but ...

$$P(s) = \frac{1}{s+1}, \quad P^{-1}(s) = s + 1$$

requires differentiation!

$$P(s) = \frac{e^{-s}}{s+1}, \quad P^{-1}(s) = (s+1)e^{s}$$

requires differentiation and prediction

$$P(s) = \frac{s-1}{s+1}, \quad P^{-1}(s) = \frac{s+1}{s-1}$$

The inverse is unstable

Approximate Inverses

Since it is difficult to obtain an exact inverse we have to approximate. One possibility to find an approximate inverse $P^{\dagger}$ of the transfer function G is to find the transfer function X(s) which minimizes

$$J = \int_0^\infty (u^2(t) - v^2(t))dt$$

where $V(s) = P(s)X(s)U(s)$ and u is a given input signal. Notice that the approximate inverse is matched to a particular input u.

Approximate Inverses

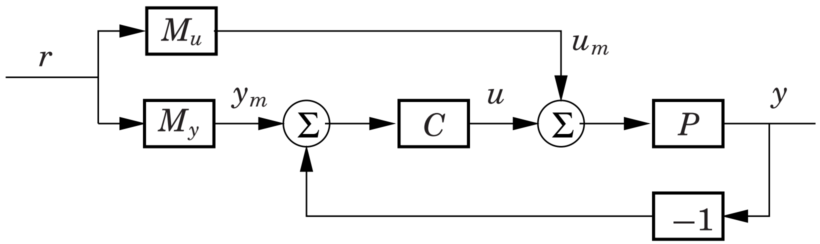
Solving an optimization problem for step signals give the following approximate inverses

$$P(s) = \frac{1}{1+s}, \quad P^{\dagger}(s) = \frac{1+s}{1+sT}$$

$$P(s) = \frac{1-s}{1+s}, \quad P^{\dagger}(s) = 1$$

$$P(s) = e^{-s}, \quad P^{\dagger}(s) = 1$$

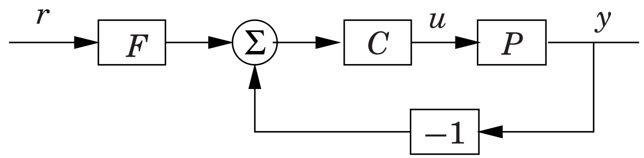
Response to Reference Signals



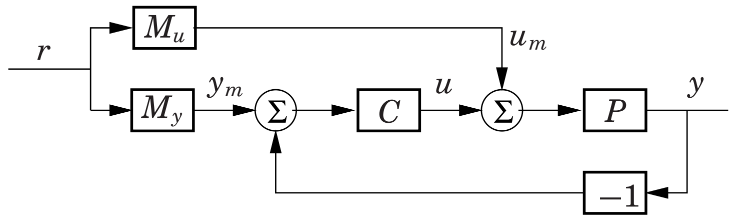
Notice that M_u and M_y can be viewed as generators of the desired output y_m and the inputs u_m which corresponds to y_m.

Assume that process P, controller C and desired response M_y are given are given. Then $M_u = M_y/P$. Does not depend on C!

4. Improved Response to Reference Signals



An alternative representation



Relation: $FC = M_yC + M_u$

Design of Feedforward

Let that process P, controller C and desired response M_y are given be given. Since $M_u = M_y/P$ and the transfer function should be stable, causal and not include derivatives we find that.

- Unstable process zeros must be zeros of M_y
- Time delays of the process must be time delays of M_y
- The pole excess of M_y must be greater than the pole excess of P

Take process limitations into account!

Example of Feedforward Design

Assume $P = \dfrac{1}{s+1}$ and $M_y = \dfrac{1}{sT+1}$ then

$$M_u = \frac{s+1}{sT+1} = \frac{1}{T}\Big(1 - \frac{1-T}{sT+1}\Big)$$

The direct term (high frequency gain of M_u) is $1/T$ which implies that fast response requires high gain of M_u. Notice

$$\frac{M_u(\infty)}{M_u(0)} = \frac{1}{T}$$

Large control signals if $T << 1$!

Example of Feedforward Design

Assume $P = \dfrac{1}{(s+1)^4}$ and $M_y = \dfrac{1}{(sT+1)^4}$ then

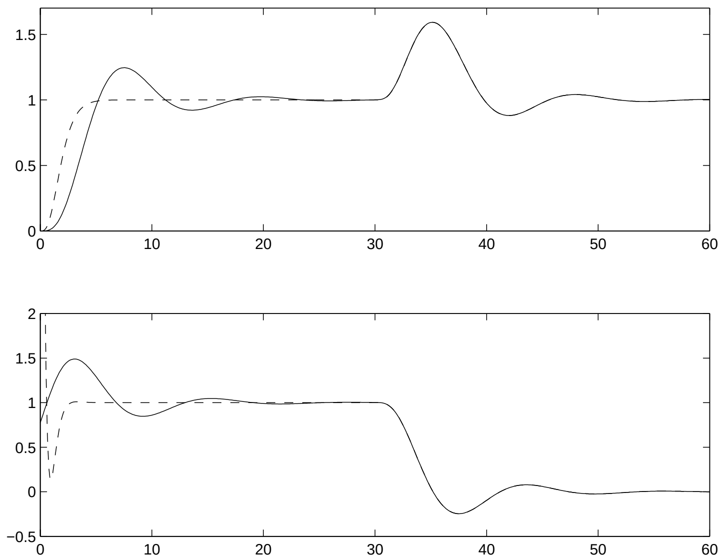
$$M_u = \frac{(s+1)^4}{(sT+1)^4}$$
$$= \frac{1}{T^4}\Big(1 - \frac{4(1-T)(sT)^3 + (6(1-T^2)(sT)^2 + (4(1-T^3)sT + 1 - T^4}{(sT+1)^4}\Big)$$

Direct term is $1/T^4$ which implies that fast response requires high gain of M_u. Notice

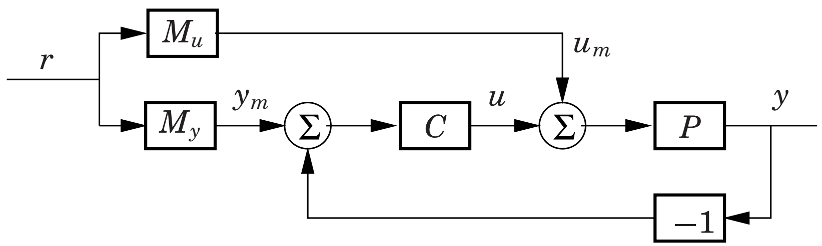
$$\frac{M_u(\infty)}{M_u(0)} = \frac{1}{T^4}$$

The high frequency gain of M_u depends strongly on the pole excess and the speed up that we require. Bounds on the control signal limit how fast response we can obtain.

Simulation (Gang of Six)



Neutral Feedforward



Response to reference signals

$$\frac{Y(s)}{R(s)} = \frac{P(CM_y + M_u)}{1 + PC} = M_y + (PM_u - M_y)S$$

Choosing $M_y = P$ $M_u = 1$ gives a system where the response to reference signals is the open loop dynamics $P(s)$.

Combined Feedback and Feedforward

Reduction of measurable disturbance

$$G_d = \frac{Y(s)}{D(s)} = \frac{P_2(1 - P_1F)}{1 + PC} = P_2(1 - P_1F)S$$

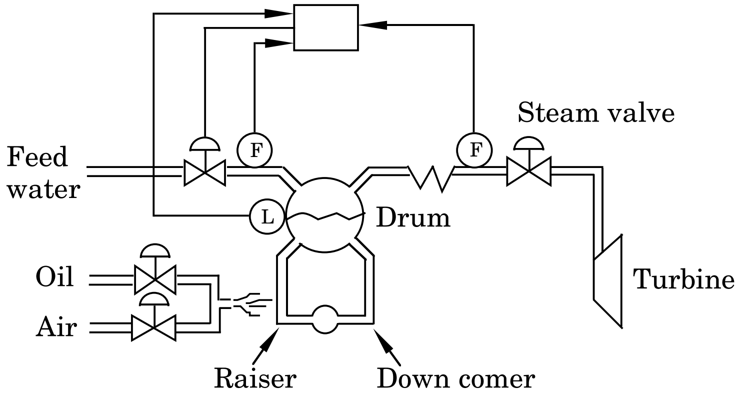
Response to reference signals

$$\frac{Y(s)}{R(s)} = \frac{P(CM_y + M_u)}{1 + PC} = M_y + (PM_u - M_y)S$$

Feedback will deal nicely with frequencies where the sensitivity is small. Feedforward only needs to act for frequencies where the sensitivity is close to one. Approximate models can be used.

Use of Feedforward

- Temperature control in buildings. Measure outside temperature and adjust heating
- Level control in steam generators. To deal with shrink and swell.



Feedback and Feedforward

- Feedback
- Closed loop
- Acts only when there are deviations
- Market Driven
- Robust to model errors $|S| < 1$ for some ω
- Risk for instability

- Feedforward
- Open loop
- Acts before deviations show up
- Planning
- Not robust to model errors $|S| = 1$ for all ω
- No risk for instability

Feedforward is a nice complement to feedback. Properly used it can improve a control system substantially. Its use is increasing. It requires good models and should always be used together with feedback.

5. Summary

- Feedforward is a useful technique.
- It can be used to improve response to reference signals and reduction of the effect of measurable disturbances
- Feedforward has poor sensitivity $S = 1$!
- Useful to combine with feedback
- Use of feedforward is increasing