CONTROLLER DESIGN EXAMPLES

(*For the QFTCT*)

See the web page of the *Control and Energy Systems Center*, at Case Western Reserve University, for details. http://cesc.case.edu

G-1 INTRODUCTION

This appendix presents a set of selected examples to help the reader to understand the main concepts and procedures of the QFT Control Toolbox (*QFTCT*), ver.1.01.

Table G.1 Examples, Controller design.

Example and File	Model	Characteristics	Specifications
1. ***DC motor*** Example1_appendixG.mat	$P(s)=\dfrac{k}{s(s+a)(s+b)}$	Stable, 3 parameters with uncertainty	Robust Stability, Reference tracking, Disturbance rejection
2. ***Satellite*** Example2_appendixG.mat	$P(s)=\dfrac{k}{s^2}$	2 integrators, 1 parameter with uncertainty	Robust Stability, Reference tracking
3. ***Two carts coupled by spring*** Example3_appendixG.mat	$P(s)=\dfrac{1}{s^2\ (a\,s^2+b)}$	2 imaginary poles, 2 integrators, 3 parameters with coupled uncertainty	Robust Stability
4. ***Inverted Pendulum*** Example4_appendixG.mat	$\dot{\mathbf{x}}=\begin{bmatrix} 0 & 1 & 0 \\ 0 & -\dfrac{k(I+mL^2)}{(M+m)I+mL^2M} & -\dfrac{m^2L^2g}{(M+m)I+mL^2} \\ 0 & 0 & 0 \\ 0 & \dfrac{mLk}{(M+m)I+mL^2M} & \dfrac{mLg(M+m)}{(M+m)I+mL^2} \end{bmatrix}$ $y=[0\quad 0\quad 1\quad 0]\mathbf{x}$	Unstable, 4 parameters with uncertainty	Robust Stability, Disturbance rejection
5. ***Central heating system*** Example5_appendixG.mat	$P(s)=\dfrac{k}{\tau\,s+1}\mathrm{e}^{-L\,s}$	System with delay, 3 parameters with uncertainty	Robust Stability, Disturbance rejection

The related control theory can be found along the book, in Chaps. 2 through 8 and Apps. A through E. The *QFTCT* user's guide is in App. F. The selected examples are plants with model uncertainty, and stability and performance robust specifications. They cover a wide range of model and

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control aspects. A classification of the selected examples is shown in Table G.1. Each of the examples is broken up into two parts: P1 *Setting Up the Problem* and P2 *Solving the Problem*. The solutions of the examples with the QFT Control Toolbox are included in the folder.

G-2 DC MOTOR. (EXAMPLE 1 – P1).

DC motors are direct current machines employed extensively in industrial applications, including wind turbines. Medium and high power DC motors are often employed in wind turbines for pitch control systems (to move the blades and change the aerodynamic coefficient of the rotor) and for yaw control systems (to move the nacelle following the wind direction).

In this example, it is considered a yaw control system for a wind turbine. It consists of DC motors (the actuators), the nacelle to be oriented (inertia), the bearings (friction and inertia), a vane (sensor), and the controller. The transfer function between the angle $y(s)$ to be controlled (output) and the voltage $u(s)$ applied to the DC motors actuators is:

$$\frac{y(s)}{u(s)} = P(s) = \frac{K_m}{s(J\,s+D)(L_a\,s+R_a)} = \frac{k}{s(s+a)(s+b)} \qquad \textbf{(G.1)}$$

where J is the inertia of the rotating elements, D the viscous friction, K_m the motor-torque constant, and R_a the resistance and L_a the inductance of the motor armature. Combining these parameters, it is possible to lump the parameters in $k = K_m/(L_a\,J)$ as the gain, $1/a = J/D$ as the mechanical time constant and $1/b = L_a/R_a$ as the electrical time constant of the system. For a very small wind turbine, the parameters and the associate uncertainty for the system are: $610 \le k \le 1050$; $1 \le a \le 15$; $150 \le b \le 170$.

The problem is to synthesize a compensator $G(s)$ and a pre-filter $F(s)$ for the DC motor defined above, to achieve the following specifications:

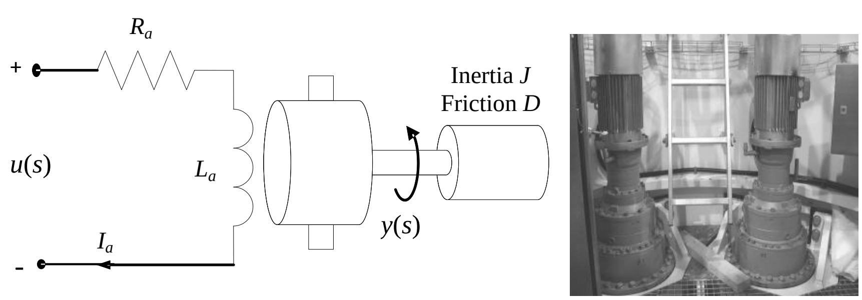

Fig. G.1 DC motor and a yaw wind turbine system.

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(1) Robust Stability. A minimum phase margin angle of 40º.

(2) Reference Tracking. Given by tracking models $\delta_{upper}(s)$ and $\delta_{lower}(s)$ to satisfy: M_p = 1.30 and t_s = 1.92 s (for the upper bound), and t_s = 1.70 s (for the lower bound).

(3) Disturbance rejection at plant input. To satisfy: $\alpha_p < |y_D(t_p)| = 0.1$ for t_p = 65 ms and for a unit disturbance step input. $y_D(\infty) = 0$, $\omega < 10$ rad/s

G-3 SATELLITE. (EXAMPLE 2 – P1).

This example deals with the design of controllers to regulate the relative distance (dx, dy, dz) between two spacecraft flying in formation in deep space and with no ground intervention. The second spacecraft is fixed, while the first spacecraft moves the thrusters (u_{x1}, u_{y1}, u_{z1}) to control the relative distance (dx, dy, dz) between both units, which is measured by RF and Laser based metrology. The system is described as,

$$dx = x_2 - x_1 \,; \quad dy = y_2 - y_1 \,; \quad dz = z_2 - z_1 \,; \tag{G.2}$$

$$\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} p_{1x}(s) & 0 & 0 \\ 0 & p_{1y}(s) & 0 \\ 0 & 0 & p_{1z}(s) \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{z1} \end{bmatrix} \tag{G.3}$$

which presents three independent SISO systems, one for each axis (x, y, z). The plant models have uncertainty due to fuel consumption, so that,

$$P_{1x}(s) = P_{1y}(s) = P_{1z}(s) = P_1(s) = \frac{-1}{m_1 s^2}, \quad m_1 \in [360,\, 460] \text{ kg} \tag{G.4}$$

Synthesize the compensator $G(s)$ and the pre-filter $F(s)$ for the uncertain plant of the x axis of the first spacecraft (Fig. G.2), and for the following specifications:

(1) Robust Stability. $\left| \dfrac{P_1(j\omega)G(j\omega)}{1+P_1(j\omega)G(j\omega)} \right| \le 1.1 \quad \forall \omega$ which involves a phase

margin of at least 55º and a gain margin of at least 1.99 (5.9 dB).

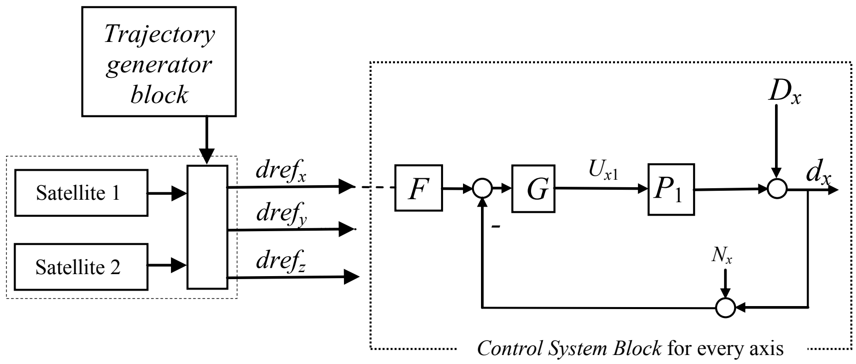

Fig. G.2 Spacecraft control system.

(2) <u>Reference tracking</u>. $T_{R_L}(\omega) \le \left|\dfrac{P_1(j\omega)G(j\omega)}{1+P_1(j\omega)G(j\omega)}\right| \le T_{R_U}(\omega)$ where,

$$T_{R_U}(\omega) = \left| \frac{k\left(\dfrac{j\omega}{a}+1\right)}{\left(\dfrac{j\omega}{\omega_{n1}}\right)^2 + \dfrac{2\,\zeta}{\omega_{n1}}\,(j\omega)+1} \right| ; T_{R_L}(\omega) = \left| \frac{1}{\left(\dfrac{j\omega}{\sigma_1}+1\right)\left(\dfrac{j\omega}{\sigma_2}+1\right)\left(\dfrac{j\omega}{\sigma_3}+1\right)} \right|$$

and where, $k = 1$; $\omega_{n1} = 0.025$; $\zeta = 0.8$; $a = 0.035$; $\omega_{n2} = 0.03$; $\sigma_1 = 2\ \omega_{n2}$; $\sigma_2 = 0.3\ \omega_{n2}$; $\sigma_3 = \omega_{n2}$; for $\omega =$ [0.00001 0.00005 0.0001 0.0005 0.001 0.005 0.01 0.05 0.1 0.5 1.0 2.0 3.0] rad/s.

G-4 TWO CARTS. (EXAMPLE 3 – P1).

Consider the classical ACC benchmark problem shown in Fig. G.3. It is a mechanical frictionless system composed of two carts of mass m_1 and m_2, coupled by a link (spring) of stiffness γ. The problem is to control the position $x_2(t)$ of the second cart by applying a force $u(t)$ to the first cart.

The transfer function between the position $x_2(t)$ of the second cart and the force $u(t)$ in the first cart is:

$$\frac{x_2(s)}{u(s)} = P(s) = \frac{1}{s^2\left[(m_1\, m_2)s^2 + \gamma(m_1+m_2)\right]} \qquad \textbf{(G.5)}$$

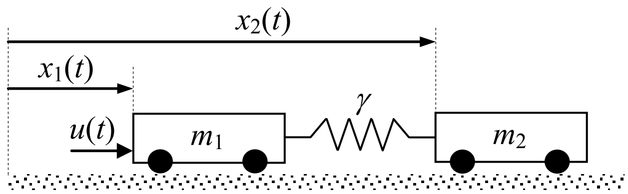

Fig. G.3 ACC benchmark problem: Two carts coupled by a spring.

with parametric uncertainty: $m_1 \in [0.9, 1.1]$; $m_2 \in [0.9, 1.1]$; $\gamma \in [0.4, 0.6]$. Synthesize the compensator $G(s)$ to control the position $x_2(t)$, fulfilling the robust stability specification: $\left|\dfrac{P(j\omega)\ G(j\omega)}{1+P(j\omega)\ G(j\omega)}\right| \le 1.2\ ,\ \ \forall\omega\in[0,\infty]$

G-5 INVERTED PENDULUM. (EXAMPLE 4 – P1).

Consider an inverted pendulum mounted on a cart, as shown in Fig. G.4. The cart is controlled so that the mass m is always in the upright position. The equations that describe the system are:

$$\begin{aligned}(M+m)\ddot{x} &= u - mL\ddot{\theta}\cos\theta + mL\dot{\theta}^2\sin\theta - k\dot{x} \\ I\ddot{\theta} &= mgL\sin\theta - mL^2\ddot{\theta} - mL\ddot{x}\cos\theta\end{aligned} \tag{G.6}$$

where θ is the pendulum angle to be controlled, u the force to be applied to the cart, x the linear position of the cart, L the stick length, M the cart mass, m the end mass, I the stick inertia, and k the friction coefficient between the cart and the rail. The parameters are:

$$\begin{aligned}&m\in[0.1,0.2]\,kg;\ \ M\in[0.9,1.1]\,kg;\ \ I\in[0.005,0.01]\,kg\,m^2;\\ &k\in[0.1,0.2]\,Ns/m;\ \ L=0.5\ m;\ \ g=9.8\ m/s^2\end{aligned}$$

By linearizing the system around the equilibrium point, $\theta(t) = 0$ and $d\theta(t)/dt = 0$, the equations are,

$$\begin{aligned}(M+m)\ddot{x} + mL\ddot{\theta} + k\dot{x} - u &= 0 \\ (I+mL^2)\ddot{\theta} + mL\ddot{x} - mgL\theta &= 0\end{aligned} \tag{G.7}$$

Given the state variables: $\boldsymbol{x} = [x_1\ \ x_2\ \ x_3\ \ x_4]^T = [x\ \ \dot{x}\ \ \theta\ \ \dot{\theta}]^T$

then the state space description is given by:

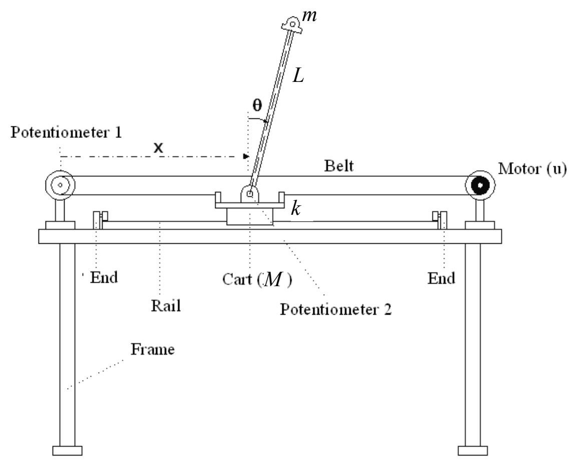

Fig. G.4 Inverted pendulum.

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\dfrac{k\left(I+mL^2\right)}{(M+m)I+mL^2M} & -\dfrac{m^2L^2g}{(M+m)I+mL^2M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{mLk}{(M+m)I+mL^2M} & \dfrac{mLg(M+m)}{(M+m)I+mL^2M} & 0 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ \dfrac{I+mL^2}{(M+m)I+mL^2M} \\ 0 \\ -\dfrac{mL}{(M+m)I+mL^2M} \end{bmatrix} u \tag{G.8}$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}\boldsymbol{x}$$

Design a compensator $G(s)$ to keep the pendulum in the upright position for the following stability and performance specifications:

(1) Robust Stability. A minimum phase margin of 45º.

(2) Disturbance rejection at plant output:

$$\left|\frac{1}{1+P(j\omega)G(j\omega)}\right| < \left|\frac{0.05\left(\dfrac{j\omega}{0.05}+1\right)}{(j\omega+1)}\right|$$

G-6 CENTRAL HEATING SYSTEM. (EXAMPLE 5 – P1).

Consider a central heating system of a three floor building (Fig. G.5). The transfer function that describes the inner room temperature $T_r(t)$ in terms of the desired mixed water temperature $T_{md}(t)$, that comes from the mixing valve, is linearized around the equilibrium point, and is:

$$\frac{T_r(s)}{T_{md}(s)} = P(s) = \frac{k}{\tau\, s+1}\mathrm{e}^{-L\,s} \qquad \textbf{(G.9)}$$

Using system identification techniques with experimental data, the obtained parameter are: $k \in [40, 60]$; $\tau \in [800, 1200]$ sec.; $L \in [100, 200]$ sec.

Design a loop compensator $G(s)$ so that the system fulfills the following specifications:

(1) Robust Stability. $\left|\dfrac{P(j\omega)\ G(j\omega)}{1+P(j\omega)\ G(j\omega)}\right| \le 1.2$

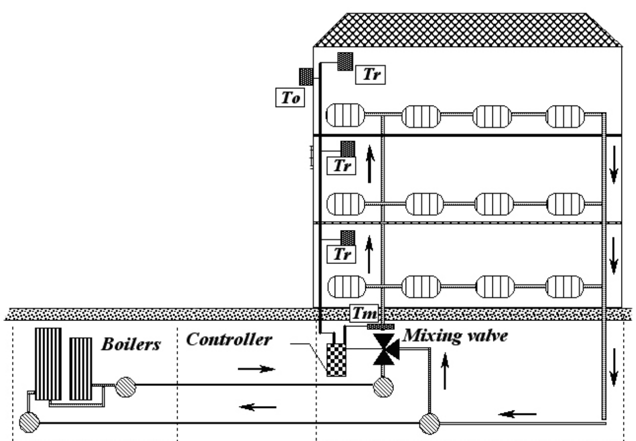

Fig. G.5 Central heating system.

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for ω = [0.0001 0.0003 0.0005 0.0007 0.001 0.003 0.005 0.007 0.01 0.03 0.05 0.07 0.1] rad/s

(2) Disturbance rejection at plant input

$$\left| \frac{P(j\omega)}{1+P(j\omega)\,G(j\omega)} \right| \le 0.3$$

for ω = [0.0001 0.0003 0.0005 0.0007] rad/s

G-7 CONTROLLER DESIGN SOLUTIONS

Some possible solutions for the above examples, calculated by using the QFT Control Toolbox (*QFTCT*), are shown in Table G.2.

Table G.2 Controller design solutions.

	$G(s)$	$F(s)$
1. *DC motor*	$G(s)=\dfrac{89\left(\frac{s}{10}+1\right)\left(\frac{s}{1.5}+1\right)}{s\left(\frac{s}{300}+1\right)}$	$F(s)=\dfrac{\left(\frac{s}{50}+1\right)\left(\frac{s}{100}+1\right)}{\left(\frac{s}{4}+1\right)\left(\frac{s}{8}+1\right)}$
2. *Satellite*	$G(s)=\dfrac{-5\times10^{-8}\left(\frac{s}{1.16\times10^{-8}}+1\right)}{\left(\frac{s}{0.014}+1\right)}$	$F(s)=\dfrac{6.408e5s^3+1.632e4s^2+209.6s+1}{1.143e5s^3+9371s^2+202.9s+1}$
3. *Two carts coupled by spring*	$G(s)=\dfrac{\left(\frac{s}{0.5}+1\right)\left(4s^2+0.8s+1\right)}{\left(\frac{s}{12}+1\right)\left(\frac{s}{100}+1\right)\left(\frac{s}{500}+1\right)}$	None
4. *Inverted Pendulum*	$G(s)=\dfrac{-41.2\left(\frac{s}{0.6}+1\right)\left(\frac{s}{5}+1\right)}{s\left(\frac{s}{2400}+1\right)}$	None
5. *Central heating system*	$G(s)=\dfrac{7.2\times10^{-5}\left(\frac{s}{0.0013}+1\right)\left(\frac{s}{0.02}+1\right)\left(\frac{s}{2.3}+1\right)}{s\left(\frac{s}{1.1}+1\right)\left(\frac{s}{1.1}+1\right)}$	None

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